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MATHÉMATIQUES

SUR LE PROBLÈME DE LA DÉFORMATION DES CONGRUENCES RECTILIGNES AVEC CONSERVATION DE LEURS SURFACES DISTRIBUTIVES

Par S. ROSSINSKI

(Présenté par S. A. Tchapliguine, de l'Académie, le 17. III. 1938)

§ 1. Dans l'article «Déformation d'une congruence rectiligne avec conservation des surfaces distributives» (1) j'ai posé le problème général sur la déformation d'une congruence rectiligne avec conservation des surfaces distributives et j'ai indiqué la solution dans le cas d'une congruence dont les rayons sont orthogonaux aux plans tangents correspondants de la surface S au cours de la déformation de S.

Le but de cette Note est l'étude des congruences C dont les rayons se trouvent dans les plans tangents correspondants de la surface S pendant la déformation de S.

Le cas d'une déformation arbitraire de S ne conduit qu'aux surfaces développables S ou bien aux congruences dégénérées C et, par suite, il est privé d'intérêt.

Au contraire, considérant la déformation continue de la surface S à réseau conjugué persistant (déformation à base principale) on trouve quelques résultats assez généraux.

§ 2. Supposons que la surface S soit douée d'une base principale et rapportons-la à cette base.

Soient $\bar{\rho}(u, v)$, $\bar{n}(u, v)$ le vecteur définissant S et le vecteur unitaire de sa normale. Le carré de l'élément linéaire et la seconde forme quadratique de la surface S auront la forme:

$$d\bar{\rho}^2 = E du^2 + 2F du dv + G dv^2,$$
$$-d\bar{\rho}\, d\bar{n} = D du^2 + D'' dv^2.$$

Désignons par l la longueur du segment porté sur le rayon de la congruence C par les tangentes aux courbes $u=\text{const}$ et $v=\text{const}$ de la base principale de S et par $\bar{\zeta}$ le vecteur unitaire définissant ce rayon-même.

Soient, ensuite, ξ et η les longueurs des segments portés sur les tangentes aux courbes $v=\text{const}$ et $u=\text{const}$ respectivement par le rayon indiqué de C; alors, on obtient:

$$\bar{\zeta} = \frac{1}{l}\left(\frac{\eta \bar{\rho}_v}{\sqrt{G}} - \frac{\xi \bar{\rho}_u}{\sqrt{E}}\right) = a\bar{\rho}_u + b\bar{\rho}_v.$$

Supposons que la surface de départ de la congruence C est définie par le vecteur:

$$\bar{\rho}_1(u, v) = \bar{\rho} + \frac{\bar{\xi}\bar{\rho}_u}{\sqrt{E}} = \bar{\rho} + c\bar{\rho}_u$$

et que les formes fondamentales de Kummer attachées à C sont:

$$d\bar{\zeta}^2 = E_0 du^2 + 2F_0 du dv + G_0 dv^2,$$
$$d\bar{\zeta} d\bar{\rho}_1 = e du^2 + (f + f') du dv + g dv^2.$$

L'équation différentielle des surfaces distributives de C:

$$[E_0(E_0 g - G_0 e) - F_0(E_0 f + E_0 f' - 2F_0 e)]\, du^2 +$$
$$+ 2[F_0(E_0 g + G_0 e) - E_0 G_0 (f + f')]\, du dv +$$
$$+ [G_0(G_0 e - E_0 g) - F_0(G_0 f + G_0 f' - 2F_0 g)]\, dv^2 = 0$$

en vertu de l'équation de Gauss

$$DD'' = KH^2$$

(K étant la courbure de S, $H^2 = EG - F^2$) aura dans le cas actuel la forme:

$$D^8 \lambda du^2 + D^6(\lambda_1 du^2 + 2\mu_1 du dv + \nu_1 dv^2) +$$
$$+ D^4(\lambda_2 du^2 + 2\mu_2 du dv + \nu_2 dv^2) +$$
$$+ D^2(\lambda_3 du^2 + 2\mu_3 du dv + \nu_3 dv^2) + \nu_4 dv^2 = 0,$$

$\lambda, \lambda_1, \mu_1, \ldots, \nu_4$ étant des fonctions déterminées des valeurs E, F, G, ξ, η et de leurs dérivées par rapport à u et v.

Or, l'équation précédente ne peut être vérifiée au cours de la déformation continue de S sans se réduire à une identité; par suite:

$$\lambda du^2 = 0,$$
$$\lambda_1 du^2 + 2\mu_1 du dv + \nu_1 dv^2 = 0,$$
$$\lambda_2 du^2 + 2\mu_2 du dv + \nu_2 dv^2 = 0,$$
$$\lambda_3 du^2 + 2\mu_3 du dv + \nu_3 dv^2 = 0,$$
$$\nu_4 dv^2 = 0.$$

Ces équations doivent se réduire à une seule et celle-ci nous donnera les surfaces distributives de la congruence C qui sont conservées au cours de la déformation de S.

Écartons le cas privé d'intérêt des surfaces distributives confondues, c'est-à-dire supposons dans tout ce qui suit

$$\lambda = \nu_4 = 0.$$

Rejetons aussi le cas banal des surfaces développables S. Parmi les facteurs qui se trouvent dans les expressions des coefficients λ et ν_4 on a, en particulier, la valeur a dans λ et la valeur b dans ν_4. Par conséquent, parmi les différents cas nécessaires à examiner on aura les cas où $a = 0$ où bien $b = 0$.

§ 3. Le cas $a = 0$.

Dans ce cas, ξ n'étant pas nul, nous avons $l = \eta = \infty$. On trouve alors que S doit être une surface arbitraire douée d'une base principale comprenant une famille de géodésiques ($u = \text{const}$).

Les surfaces distributives de la congruence C correspondent au réseau des lignes de la base principale. Le rayon de la congruence C est paral-

lèle à la tangente de la géodésique $u=\text{const}$ et découpe sur la tangente à la courbe $v=\text{const}$ le segment qui est défini par la formule:

$$\xi = e^{\int \frac{F}{E}\left\{\begin{smallmatrix}11\\2\end{smallmatrix}\right\} du}\left[C(v) - \int \sqrt{E} e^{-\int \frac{F}{E}\left\{\begin{smallmatrix}11\\2\end{smallmatrix}\right\} du} du\right],$$

$C(v)$ étant une fonction arbitraire de v.

Dans le cas actuel nous avons $F_0 = e = g = 0$, par suite, la surface de départ de C coïncide avec sa surface moyenne.

Ainsi, la surface moyenne de la congruence est le lieu des points d'intersection des rayons de C avec les tangentes correspondantes des courbes $v=\text{const}$ de la base principale de S.

On voit qu'au cours de la déformation de la surface S la surface moyenne de C reste surface moyenne de la congruence C transformée.

Ensuite, f et f' ne dépendent pas de D et D'' et, d'autre part, on a:

$$E_0 G_0 - F_0^2 = \left[\frac{HD''}{G\sqrt{G}}\left\{\begin{smallmatrix}12\\1\end{smallmatrix}\right\}\right]^2,$$

donc les foyers et les points-limites ne sont pas conservés au cours de la déformation, mais, au contraire, se rapprochent ou se dispersent sur chaque rayon.

Notons, enfin, que la congruence C n'est pas une congruence de normales.

Supposons, en particulier, que la surface S soit une surface arbitraire de Voss. Dans ce cas on construit la congruence C tout à fait simplement, le segment ξ étant défini par la formule

$$\xi = -s_v + V(v),$$

où s_v désigne l'arc de la géodésique $v=\text{const}$ et $V(v)$ est une fonction arbitraire de v.

La quantité a peut s'annuler aussi bien en vertu de l'égalité $\xi = 0$; or, cette hypothèse ne donne que des congruences dégénérées (cylindriques, $E_0 G_0 - F_0^2 = 0$).

Le cas $b=0$. Ce cas est équivalent au cas $\xi = 0$.

§ 4. Les quantités a et b étant simultanément différentes de zéro, le problème posé ne donne que des solutions banales.

Pareillement, l'étude du cas d'un rayon de C passant par le point de contact et différent de la tangente à la courbe de la base principale ne conduit qu'aux solutions banales.

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LITTÉRATURE CITÉE

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MATHEMATIK

EINE VERALLGEMEINERUNG DER THEORIE DER REIN UNSTETIGEN STOCHASTISCHEN PROZESSE VON W. FELLER

Von W. DUBROVSKI

(Vorgelegt von S. N. Bernstein, Mitglied der Akademie, d. 25. III. 1938)

Das Ziel dieser Note ist die Entwicklung der Fellerschen Theorie der rein unstetigen stochastischen Prozesse (1) in dem allgemeinen Falle, der den Definitionen und den Problemstellungen der Arbeit von A. Kolmogoroff: «Analytische Methoden der Wahrscheinlichkeitsrechnung» (2) entspricht.

Es sei A die Menge aller möglichen Zustände eines sich verändernden Systems und $\mathfrak{M}$—die Menge aller Untermengen von A, die beliebige Differenzen und endliche oder abzählbare Summen ihrer Elemente, sowie einzelne Elemente aus A, die Menge A selbst und die leere Menge enthält. Die im folgenden betrachteten Integrale sind im Sinne von Lebesgue-Stieltjes zu verstehen und beziehen sich auf $\mathfrak{M}$.

§ 1. Es sei $\overline{p}(t,x,\tau)$ die Wahrscheinlichkeit dafür, dass das betrachtete System, das sich zur Zeit t im Zustande x befindet, sich in der Zeit $t \leqslant t' < \tau$ irgendwie verändert, und $\overline{P}(t,x,\tau,\mathfrak{E})$ die Wahrscheinlichkeit dafür, dass dieses System sich zur Zeit τ in einem der Zustände befindet, die der Menge $\mathfrak{E}$ angehören, wobei die Veränderung sich in der Zeit $t \leqslant t' < \tau$ vollzogen haben muss. Es mögen ferner die beiden Wahrscheinlichkeiten folgendermassen darstellbar sein:

$$\overline{p}(t,x,\tau) = p(t,x)(\tau - t) + \varepsilon_1, \quad \overline{P}(t,x,\tau,\mathfrak{E}) = P(t,x,\mathfrak{E}) + \varepsilon_2,$$

wo die gegebene Funktion $p(t,x)$ stetig in t und messbar in x ist, und den folgenden Ungleichungen genügt: $0 \leqslant p(t,x) \leqslant K(T)$, und die gegebene Funktion $P(t,x,\mathfrak{E})$ stetig in t, messbar in x und volladditiv bezüglich $\mathfrak{E}$ ist, und den folgenden Relationen genügt: $0 \leqslant P(t,x,\mathfrak{E}) \leqslant P(t,x,A) = 1$; die Funktion $\overline{p}(t,x,\tau)$ ist messbar in x; die Funktion $\overline{P}(t,x,\tau,\mathfrak{E})$ ist messbar in x und volladditiv bezüglich $\mathfrak{E}$; wenn

$$(\tau - t) \to 0, \quad \text{dann} \quad \frac{\varepsilon_1}{\tau - t} \to 0, \quad \varepsilon_2 \to 0, \quad \left|\frac{\varepsilon_1}{\tau - t}\right| < M = \text{const}, \quad \underset{(A)}{\text{var}}\, \varepsilon_2 < M,$$

wobei alle aufgezählten Bedingungen und Abschätzungen sich auf beliebige t, x, τ und $\mathfrak{E}$ beziehen, die den Bedingungen $-T < t < \tau < T = \text{const}$, $x \subset A$, $\mathfrak{E} \subset \mathfrak{M}$ genügen.

Es ist leicht einzusehen, dass die Wahrscheinlichkeit $F(t, x, \tau, \mathfrak{E})$ dafür, dass das System, das sich zur Zeit t im Zustande x befunden hat, zur Zeit τ sich in einem Zustande befinden wird, der der Menge $\mathfrak{E}$ angehört, durch

$$F(t, x, \tau, \mathfrak{E}) = [1 - p(t, x)(\tau - t)] E(x, \mathfrak{E}) + p(t, x) P(t, x, \mathfrak{E})(\tau - t) + \varepsilon \quad (1)$$

gegeben ist, wo $E(x, \mathfrak{E}) = 1$, falls $x \subset \mathfrak{E}$, $E(x, \mathfrak{E}) = 0$, falls $x \subset A - \mathfrak{E}$,

$$\varepsilon = [P(t, x, \mathfrak{E}) - E(x, \mathfrak{E})] \varepsilon_1 + p(t, x)(\tau - t) \varepsilon_2 + \varepsilon_1 \varepsilon_2.$$

Wenn wir in der Gleichung

$$F(t, x, \tau, \mathfrak{E}) = \int_A F(t, x, s, dA_y) F(s, y, \tau, \mathfrak{E}) \quad (t < s < \tau) \quad (2)$$

den Faktor $F(s, y, \tau, \mathfrak{E})$ mit Hilfe von (1) ausdrücken, und bemerken, dass

$$\int_A F(t, x, s, dA_y) E(y, \mathfrak{E}) = F(t, x, s, \mathfrak{E})$$

ist, so bekommen wir ohne weiteres die folgende fundamentale Beziehung:

$$\frac{\partial F(t, x, \tau, \mathfrak{E})}{\partial \tau} = -\int_{\mathfrak{E}} F(t, x, \tau, dA_y) p(\tau, y) +$$

$$+ \int_A F(t, x, \tau, dA_y) p(\tau, y) P(\tau, y, \mathfrak{E}). \quad (3)$$

Wenn wir andererseits in (2) den Faktor $F(t, x, s, dA_y)$ mit Hilfe von (1) ausdrücken, und bemerken, dass

$$\int_A E(x, dA_y) F(s, y, \tau, \mathfrak{E}) = F(s, x, \tau, \mathfrak{E})$$

ist, so können wir uns leicht von der Gültigkeit der zweiten fundamentalen Beziehung

$$\frac{\partial F(t, x, \tau, \mathfrak{E})}{\partial t} = p(t, x) \left[F(t, x, \tau, \mathfrak{E}) - \int_A P(t, x, dA_y) F(t, y, \tau, \mathfrak{E}) \right] \quad (4)$$

überzeugen. Ausser der Gleichungen (3) und (4) genügt die Funktion $F(t, x, \tau, \mathfrak{E})$ offenbar auch den folgenden Bedingungen:

$$\lim_{\tau \to t_+} F(t, x, \tau, \mathfrak{E}) = E(x, \mathfrak{E}), \quad (5)$$

$$\lim_{t \to \tau_-} F(t, x, \tau, \mathfrak{E}) = E(x, \mathfrak{E}). \quad (6)$$

Unsere Hauptaufgabe besteht nun im Beweis der zwei folgenden Sätze:

Satz I. *Unter den gemachten Voraussetzungen sind die Wahrscheinlichkeiten $F(t, x, \tau, \mathfrak{E})$ eindeutig durch die Funktionen $p(t, x)$ und $P(t, x, \mathfrak{E})$ bestimmt.*

Satz II. *Wenn die Funktionen* $p(t,x)$, $P(t,x,\mathfrak{E})$ *den oben aufgezählten Bedingungen genügen, aber sonst beliebig gegeben sind, so existiert ein stochastischer Prozess* (im Sinne der zitierten Arbeit von A. Kolmogoroff), *der den vorgegebenen* $p(t,x)$ *und* $P(t,x,\mathfrak{E})$ *entspricht.*

Um diese beiden Sätze zu beweisen, genügt es unter den bezüglich $p(t,x)$ und $P(t,x,\mathfrak{E})$ gemachten Voraussetzungen zu beweisen, dass

1) die Gleichung (3) mit den Anfangsbedingungen (5) eine einzige Lösung $F(t,x,\tau,\mathfrak{E})$ besitzt, die den folgenden, von wahrscheinlichkeitstheoretischem Standpunkte aus, natürlichen Bedingungen genügt:

$$F(t,x,\tau,\mathfrak{E}) \geqslant 0,\quad F(t,x,\tau,A)=1;$$

2) diese Lösung auch den Gleichungen (1), (2), (4) und (6) genügt.

§ 2. Zum Existenzbeweis der Lösung von (3), die der Bedingung (5) genügt, betrachten wir die Funktionenfolge $\varphi_{k,n}(t,x,\tau,\mathfrak{E})$ $(-T<t<\tau<T;\ x\subset A;\ \mathfrak{E}\subset\mathfrak{M};\ k=0,\pm1,\pm2,\ldots;\ n=0,1,2,\ldots)$, die folgendermassen definiert ist:

$$\left.\begin{aligned} &\varphi_{0,0}=E(x,\mathfrak{E}),\quad \varphi_{k,0}=0 \quad \text{für} \quad k\neq 0,\\ &\varphi_{k,n+1}(t,x,\tau,\mathfrak{E})=\int_t^\tau d\tau\left[\int_{\mathfrak{E}}\varphi_{k,n}(t,x,\tau,dA_y)\,p(\tau,y)+\right.\\ &\qquad\left.+\int_A \varphi_{k-1,n}(t,x,\tau,dA_y)\,p(\tau,y)\,P(\tau,y,\mathfrak{E})\right]. \end{aligned}\right\} \tag{7}$$

Die Induktionsmethode ergibt leicht, dass alle Funktionen $\varphi_{k,n}$ in t und τ stetig, in x messbar und bezüglich $\mathfrak{E}$ volladditiv sind, und den folgenden Abschätzungen genügen: $\varphi_{k,n}=0$, wenn $k<0$ oder $k>n$,

$$0\leqslant\varphi_{k,n}\leqslant C_k^n\frac{[K(T)(\tau-t)]^n}{n!},\quad \text{wenn}\quad 0\leqslant k\leqslant n.$$

Wenn wir mit $F(t,x,\tau,\mathfrak{E})$ die Summe der Doppelreihe

$$\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}(-1)^{k+n}\varphi_{k,n}(t,x,\tau,\mathfrak{E})$$

bezeichnen, die offenbar, absolut und gleichmässig konvergiert, so finden wir leicht, dass die Funktion $F(t,x,\tau,\mathfrak{E})$ dieselben Eigenschaften besitzt, wie die Funktionen $\varphi_{k,n}$, wobei

$$\operatorname*{var}_{(\mathfrak{E})}F(t,x,\tau,\mathfrak{E})\leqslant\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\varphi_{k,n}(t,x,\tau,\mathfrak{E})\leqslant e^{2K(T)(\tau-t)}$$

ausfällt. Die Funktion $F(t,x,\tau,\mathfrak{E})$ stellt nun die gesuchte Lösung der Gleichung (3) dar, wie man sich ohne Mühe überzeugt, wenn man die beiden Seiten der dritten Gleichung (7) mit $(-1)^{k+n}$ multipliziert, und dann die so entstandenen Gleichungen für $k=0,1,2,\ldots;\ n=0,1,2,\ldots$ gliedweise summiert.

Offenbar ist

$$F(t, x, \tau, \mathcal{E}) = \sum_{n=0}^{\infty} F_n(t, x, \tau, \mathcal{E}), \tag{8}$$

wo $F_n(t, x, \tau, \mathcal{E}) = \sum_{k=0}^{\infty} (-1)^{n+k} \varphi_{k,n}(t, x, \tau, \mathcal{E})$ ist, woraus

$$\left.\begin{aligned} F_0(t, x, \tau, \mathcal{E}) &= E(x, \mathcal{E}), \\ F_{n+1}(t, x, \tau, \mathcal{E}) &= \int_t^{\tau} d\tau \Big[- \int_{\mathcal{E}} F_n(t, x, \tau, dA_y)\, p(\tau, y) + \\ &\quad + \int_A F_n(t, x, \tau, dA_y) p(\tau, y)\, P(\tau, y, \mathcal{E}) \Big] \end{aligned}\right\} \tag{8'}$$

folgt. Andererseits ist

$$F(t, x, \tau, \mathcal{E}) = \sum_{k=0}^{\infty} \psi_k(t, x, \tau, \mathcal{E}), \tag{9}$$

wo $\psi_k(t, x, \tau, \mathcal{E}) = \sum_{n=0}^{\infty} (-1)^{k+n} \varphi_{k,n}(t, x, \tau, \mathcal{E})$ ist, woraus

$$\left.\begin{aligned} \psi_0(t, x, \tau, \mathcal{E}) &= E(x, \mathcal{E}) - \int_t^{\tau} d\tau \int_{\mathcal{E}} \psi_0(t, x, \tau, dA_y)\, p(\tau, y), \\ \psi_k(t, x, \tau, \mathcal{E}) &= \int_t^{\tau} d\tau \Big[- \int_{\mathcal{E}} \psi_k(t, x, \tau, dA_y)\, p(\tau, y) + \\ &\quad + \int_A \psi_{k-1}(t, x, \tau, dA_y)\, p(\tau, y)\, P(\tau, y, \mathcal{E}) \Big] \end{aligned}\right\} \tag{9'}$$

folgt. Aus (8) und (8′) folgt, dass $F(t, x, \tau, A) = 1$ ist; wie wir sofort zeigen werden, folgt aus (9) und (9′), dass alle Funktionen ψ_k und die Funktion $F(t, x, \tau, \mathcal{E})$ nicht-negativ sind.

Betrachten wir zunächst die Gleichung

$$u(\tau, \mathcal{E}) = - \int_t^{\tau} d\tau \int_{\mathcal{E}} u(\tau, dA_y)\, p(\tau, y) + \int_t^{\tau} h(\tau, \mathcal{E})\, d\tau + g(\mathcal{E}), \tag{10}$$

worin $h(\tau, \mathcal{E})$ und $g(\mathcal{E})$ volladditive Funktionen sind, wobei die erste von ihnen im Intervall $|\tau| < T$ stetig in τ ist, und ihre Schwankung auf der Menge A in diesem Intervall beschränkt ist. Die Lösung dieser Gleichung kann mit der Methode der successiven Approximationen gefunden werden, aber durch unmittelbare Verifikation kann leicht festgestellt werden, dass diese Lösung durch

$$u(\tau, \mathcal{E}) = \int_{\mathcal{E}} e^{-\int_t^{\tau} p(s, y)\, ds} H(\tau, dA_y) \tag{11}$$

gegeben ist, wo

$$H(\tau, \mathcal{E}) = \int_t^\tau d\sigma \int_{\mathcal{E}} e^{\int_t^\sigma p(s,y)\,ds} h(\sigma, dA_y) + g(\mathcal{E}) \tag{11'}$$

ist. Dabei wird die folgende, leicht zu beweisende Formel benutzt:

$$\int_{\mathcal{E}} \varphi(x) f(x) \Phi(d\mathcal{E}_x) = \int_{\mathcal{E}} \varphi(x) \Big[\int_{d\mathcal{E}_x} f(x) \Phi(d\mathcal{E}_x) \Big]. \tag{12}$$

Die Einzigkeit dieser Lösung wird mit der Methode der successiven Approximationen bewiesen.

Indem wir nun die Gleichungen (9') als einen Spezialfall der Gleichung (10) betrachten, bekommen wir mit Rücksicht auf (11) und (11')

$$\left.\begin{aligned} \psi_0(t, x, \tau, \mathcal{E}) &= E(x, \mathcal{E})\, e^{-\int_t^\tau p(s,x)\,ds} \\ \psi_k(t, x, \tau, \mathcal{E}) &= \int_{\mathcal{E}} H_k(\tau, dA_y)\, e^{-\int_t^\tau p(s,y)\,ds}, \end{aligned}\right\} \tag{13}$$

wo

$$H_k(\tau, \mathcal{E}) = \int_t^\tau d\sigma \int_{\mathcal{E}_y} e^{\int_t^\sigma p(s,y)\,ds} \int_{A_z} \psi_{k-1}(t, x, \sigma, dA_z)\, p(\sigma, z)\, P(\sigma, z, dA_y) \tag{13'}$$

ist, woraus in der Tat $\psi_k \geqslant 0$, $F \geqslant 0$ folgt.

Dass die so erhaltene Lösung $F(t, x, \tau, \mathcal{E})$ der Gleichungen (3) und (5) zwischen den Lösungen, die den Bedingungen $F(t, x, \tau, \mathcal{E}) \geqslant 0$, $F(t, x, \tau, A) = 1$ genügen, oder sogar zwischen allen Lösungen, deren totale Schwankung auf $\mathcal{E}$ beschränkt ist, einzig ist, kann leicht mit Hilfe der Methode bewiesen werden, die in ähnlichen Fällen in der Theorie der Differentialgleichungen üblicherweise angewandt wird.

Ebenso leicht kann geschlossen werden, dass die Lösung $F^*(t, x, \tau, \mathcal{E})$ der Gleichung (4), die der Bedingung (6) genügt, durch

$$F^*(t, x, \tau, \mathcal{E}) = \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} (-1)^{k+n} \varphi^*_{k,n}(t, x, \tau, \mathcal{E})$$

gegeben ist, wo $\varphi^*_{0,0} = E(x, \mathcal{E})$, $\varphi^*_{k,0} = 0$ für $k \neq 0$,

$$\varphi^*_{k,n+1}(t, x, \tau, \mathcal{E}) = \int_t^\tau dt\, p(t, x) \Big[\varphi^*_{k,n}(t, x, \tau, \mathcal{E}) + \int_A P(t, x, dA_y)\, \varphi^*_{k-1,n}(t, y, \tau, \mathcal{E}) \Big], \tag{14}$$

wobei, wie oben, die Funktionen $\varphi^*_{k,n}$ und F^* in t und τ stetig, in x messbar und bezüglich $\mathcal{E}$ volladditiv sind, und den folgenden Abschätzungen genügen: $\varphi^*_{k,n} = 0$ fur $k < 0$ und $k > n$,

$$0 \leqslant \varphi^*_{k,n} \leqslant C_k^n \frac{[K(T)(\tau - t)]^n}{n!} \quad \text{für} \quad n \geqslant k \geqslant 0,$$

$$\operatorname*{var}_{(A)} F^*(t, x, \tau, \mathcal{E}) \leqslant e^{2K(T)(\tau - t)}.$$

Offenbar ist

$$F^*(t, x, \tau, \mathcal{E}) = \sum_{n=0}^{\infty} F_n^*(t, x, \tau, \mathcal{E}), \tag{15}$$

wo $F_n^*(t, x, \tau, \mathcal{E}) = \sum_{k=0}^{\infty} (-1)^{k+n} \varphi_{k,n}^*(t, x, \tau, \mathcal{E})$ ist, woraus

$$\left.\begin{aligned} F_0^*(t, x, \tau, \mathcal{E}) &= E(x, \mathcal{E}), \\ F_{n+1}^*(t, x, \tau, \mathcal{E}) &= \int_t^{\tau} dt\, p(t, x) \Big[- F_n^*(t, x, \tau, \mathcal{E}) + \\ &\quad + \int_A P(t, x, dA_y) F_n^*(t, y, \tau, \mathcal{E}) \Big] \end{aligned}\right\} \tag{15'}$$

folgt. Andererseits ist

$$F^*(t, x, \tau, \mathcal{E}) = \sum_{k=0}^{\infty} \psi_k^*(t, x, \tau, \mathcal{E}), \tag{16}$$

wo $\psi_k^*(t, x, \tau, \mathcal{E}) = \sum_{n=0}^{\infty} (-1)^{k+n} \varphi_{k,n}^*(t, x, \tau, \mathcal{E})$ ist, woraus

$$\left.\begin{aligned} \psi_0^*(t, x, \tau, \mathcal{E}) &= E(x, \mathcal{E}) - \int_t^{\tau} dt\, p(t, x)\, \psi_0^*(t, x, \tau, \mathcal{E}), \\ \psi_k^*(t, x, \tau, \mathcal{E}) &= \int_t^{\tau} p(t, x)\, dt \Big[- \psi_k^*(t, x, \tau, \mathcal{E}) + \\ &\quad + \int_A P(t, x, dA_y)\, \psi_{k-1}^*(t, y, \tau, \mathcal{E}) \Big] \end{aligned}\right\} \tag{16'}$$

folgt.

Wir beweisen nun, dass die Funktionen F und F^* identisch sind, wobei für sie die Beziehungen (1) und (2) erfüllt sind.

Betrachten wir die Funktionaloperationen

$$L(u) = -u_s'(s, \mathcal{E}) - \int_{\mathcal{E}} p(s, z)\, u(s, dA_z) + \int_A p(s, z)\, P(s, z, \mathcal{E})\, u(s, dA_z),$$

$$L^*(v) = v_s'(s, x) - p(s, x) \Big[v(s, x) - \int_A P(s, x, dA_z)\, v(s, z) \Big],$$

wobei es vorausgesetzt wird, dass die Funktionen $u(s, \mathcal{E})$ und $u_s'(s, \mathcal{E})$ in s stetig und bezüglich $\mathcal{E}$ volladditiv sind und ihre totale Schwankung auf $\mathcal{E}$ beschränkt ist, und dass die Funktionen $v(s, x)$ und $v_s'(s, x)$ in s stetig, in x messbar und beschränkt sind.

Dann besteht die folgende Beziehung:

$$\int_{t_1}^{t_2} ds \int_A \{ v(s, y)\, L[u(s, dA_y)] - L^*[v(s, y)]\, u(s, dA_y) \} =$$

$$= \int_A [v(t_1, y)\, u(t_1, dA_y) - v(t_2, y)\, u(t_2, dA_y)], \tag{17}$$

was man leicht einsieht, wenn man in dieselbe die Operatoren L und L^* in ihrer entwickelten Form einsetzt, die Reihenfolge der Integratiol nen vertauscht und, von der Formel (12) Gebrauch machend, partiel-integriert.

Wenn wir in der Formel (17) $u(s, \mathcal{E}) = F(t, x, s, \mathcal{E})$, $v(s, y) = F^*(s, y, \tau, \mathcal{E})$ setzen und bemerken, dass $L(F) = L^*(F^*) = 0$ ist, so bekommen wir

$$\int_A F^*(s, y, \tau, \mathcal{E})\, F(t, x, s, dA_y) = \psi(t, x, \tau, \mathcal{E}),$$

wo $t < s < \tau$ ist.

Im Grenzfalle, für $s \to \tau_-$ in einem Falle und für $s \to t_+$ in dem anderen, ergibt die letzte Gleichung

$$\psi(t, x, \tau, \mathcal{E}) = \int_A E(y, \mathcal{E})\, F(t, x, \tau, dA_y) = F(t, x, \tau, \mathcal{E}),$$

$$\psi(t, x, \tau, \mathcal{E}) = \int_A F^*(t, y, \tau, \mathcal{E})\, E(x, dA_y) = F^*(t, x, \tau, \mathcal{E}),$$

sodass in der Tat die Funktionen F und F^* zusammenfallen, wobei für sie die Beziehung (2) gilt. Schliesslich kann aus den Formeln (8) und (8′) folgern, dass für sie auch die Gleichung (1) gilt.

Die Fellersche Theorie ergibt sich als ein Spezialfall der obigen Überlegungen, wenn man unter A die Menge aller reellen Zahlen und unter $\mathfrak{M}$ die Gesamtheit aller (B)-messbaren Mengen versteht. Im anderen Spezialfall, wo die Menge A endlich oder abzählbar ist, und $\mathfrak{M}$ aus allen Untermengen von A besteht, kann man, wenn man

$$p(t, x_i) = p_i(t), \quad P(t, x_i, x_k) = P_{i,k}(t), \quad F(t, x_i, \tau, x_k) = F_{i,k}(t, \tau)$$

setzt, die erhaltenen allgemeinen Ergebnisse folgendermassen formulieren:

Die Verteilungsfunktionen $F_{i,k}(t, \tau)$ genügen den Gleichungen

$$\frac{\partial F_{i,k}(t, \tau)}{\partial \tau} = -p_k(\tau)\, F_{i,k}(t, \tau) + \sum_j F_{i,j}(t, \tau)\, p_j(\tau)\, P_{j,k}(\tau), \tag{18}$$

$$\frac{\partial F_{i,k}(t, \tau)}{\partial t} = p_i(t)\left[F_{i,k}(t, \tau) - \sum_j F_{j,k}(t, \tau)\, P_{i,j}(t) \right], \tag{19}$$

und den Bedingungen

$$\lim_{\tau \to t_+} F_{i,k}(t, \tau) = E_{i,k}, \tag{20}$$

$$\lim_{t \to \tau_-} F_{i,k}(t, \tau) = E_{i,k}, \tag{21}$$

wo $E_{i,k} = 1$ für $i = k$, $E_{i,k} 0$ für $i \neq k$.

Wie die Gleichung (18) und die Bedingung (20), so auch die Gleichung (19) und die Bedingung (21), bestimmen eindeutig die Funktionen $F_{i,k}$, die den folgenden Bedingungen genügen:

$$F_{i,k} \geqslant 0, \quad \sum_k F_{i,k} = 1, \quad F_{i,k}(t, \tau) = \sum_j F_{i,j}(t, t')\, F_{j,k}(t', \tau),$$

wo $t < t' < \tau$ ist. Diese Funktionen $F_{i,k}$ sind durch

$$F_{i,k}(t,\tau) = \sum_{n=0}^{\infty} F_{i,k}^{(n)}(t,\tau)$$

gegeben, wo $F_{i,k}^{(0)}(t,\tau) = E_{i,k}$, $F_{i,k}^{(n+1)}(t,\tau) = \int_t^\tau ds \Big[-p_k(s) F_{i,k}^{(n)}(t,s) +$

$$+ \sum_j F_{i,j}^{(n)}(t,s)\, p_j(s)\, P_{j,k}(s) \Big],$$

oder

$$F_{i,k}^{(0)}(t,\tau) = E_{i,k}\, e^{-\int_t^\tau p_i(s)\,ds},$$

$$F_{i,k}^{(n+1)}(t,\tau) = e^{-\int_t^\tau p_k(s)\,ds} \int_t^\tau d\sigma\, e^{\int_t^\sigma p_k(s)\,ds} \sum_j F_{i,j}^{(n)}(t,\sigma)\, p_j(\sigma)\, P_{j,k}(\sigma).$$

oder

$$F_{i,k}^{(0)}(t,\tau) = E_{i,k},$$

$$F_{i,k}^{(n+1)}(t,\tau) = -\int_t^\tau ds\, p_i(s) \Big[F_{i,k}^{(n)}(s,\tau) - \sum_j P_{i,j}(s)\, F_{j,k}^{(n)}(s,\tau) \Big],$$

oder, schliesslich,

$$F_{i,k}^{(0)}(t,\tau) = E_{i,k}\, e^{-\int_t^\tau p_i(s)\,ds},$$

$$F_{i,k}^{(n+1)}(t,\tau) = e^{-\int_t^\tau p_i(s)\,ds} \int_t^\tau d\sigma\, e^{\int_t^\sigma p_i(s)\,ds}\, p_i(\sigma) \sum_j P_{i,j}(\sigma)\, F_{j,k}^{(n)}(\sigma,\tau).$$

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MATHEMATIK

ÜBER DIE MITTELWERTE DER MESSBAREN FUNKTIONEN

Von B. LEWITAN

(Vorgelegt von S. N. Bernstein, Mitglied der Akademie, d. 29. III. 1938)

In der vorangehenden Note (1) habe ich auf eine Klasse von stetigen Funktionen hingewiesen, im deren Gebiete der Eindeutigkeitssatz gilt. Dabei habe ich vorausgesetzt, dass die Funktion für alle reellen λ Mittelwerte

$$a(\lambda)=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}f(x)\,e^{-i\lambda x}dx \tag{1}$$

annimmt.

In der vorliegenden Note gebe ich eine notwendige und hinreichende Bedingung dafür an, dass die Mittelwerte für alle reellen λ existieren.

Dann werde ich, indem ich einen Satz von S. Banach benutze, zeigen, wie man den Mittelwertbegriff erweitern kann, damit der Mittelwert für eine beliebige messbare Funktion existiert.

1. Wir beschränken uns auf die in jedem endlichen Intervall L-integrierbaren Funktionen, für welche

$$\overline{\lim_{T\to\infty}}\frac{1}{2T}\int_{-T}^{T}|f(x)|^2\,dx<\infty \tag{2}$$

ausfällt.

Es gilt der einfache

Satz I. *Damit die Funktion $f(x)$, welche der Relation (2) genügt, die Mittelwerte für alle reellen λ besitzt, ist es notwendig und hinreichend, dass es eine solche Folge von endlichen trigonometrischen Summen $\{S_n(x)\}$ gäbe, dass*

$$\sup_{-\infty<\lambda<\infty}\overline{\lim_{T\to\infty}}\frac{1}{2T}\left|\int_{-T}^{T}[f(x)-S_n(x)]e^{-i\lambda x}\,dx\right|\to 0 \tag{3}$$

wenn $n\to\infty$.

Beweis. Zunächst beweisen wir, dass die Bedingung hinreichend ist. Wenn (3) stattfindet, so kann man, mit einem fixierten λ und beliebigen $\varepsilon>0$, T_0 und n so gross nehmen, dass für $T>T_0$

$$\frac{1}{2T}\left|\int_{-T}^{T}[f(x)-S_n(x)]\,e^{-i\lambda x}\,dx\right|<\frac{\varepsilon}{3}$$

gilt. Da $S_n(x)$ eine endliche trigonometrische Summe ist, kann man $T', T'' > T$ so auswählen, dass

$$\left| \frac{1}{2T'} \int_{-T'}^{T'} S_n(x) e^{-i\lambda x} dx - \frac{1}{2T''} \int_{-T''}^{T''} S_n(x) e^{-i\lambda x} dx \right| < \frac{\varepsilon}{3}.$$

Dann haben wir

$$\left| \frac{1}{2T'} \int_{-T'}^{T'} f(x) e^{-i\lambda x} dx - \frac{1}{2T''} \int_{-T''}^{T''} f(x) e^{-i\lambda x} dx \right| =$$

$$= \left| \frac{1}{2T'} \int_{-T'}^{T'} [f(x) - S_n(x)] e^{-i\lambda x} dx - \frac{1}{2T''} \int_{-T''}^{T''} [f(x) - S_n(x)] e^{-i\lambda x} dx + \right.$$

$$\left. + \frac{1}{2T'} \int_{-T'}^{T'} S_n(x) e^{-i\lambda x} dx - \frac{1}{2T''} \int_{-T''}^{T''} S_n(x) e^{-i\lambda x} dx \right| <$$

$$< \frac{1}{2T'} \left| \int_{-T'}^{T'} [f(x) - S_n(x)] e^{-i\lambda x} dx \right| + \frac{1}{2T''} \left| \int_{-T''}^{T''} [f(x) - S_n(x)] e^{-i\lambda x} dx \right| +$$

$$+ \left| \frac{1}{2T'} \int_{-T'}^{T'} S_n(x) e^{-i\lambda x} dx - \frac{1}{2T''} \int_{-T''}^{T''} S_n(x) e^{-i\lambda x} dx \right| < \varepsilon.$$

Folglich existiert der Mittelwert $a(\lambda)$. Da λ beliebig war, existieren die Mittelwerte $a(\lambda)$ für alle reellen λ.

Nun zeigen wir, dass wenn die Mittelwerte für alle reellen λ existieren und (2) gilt, so gilt auch (3).

In der Tat, infolge von (2) kann nur eine abzählbare Menge der Werte λ existieren, für welche $a(\lambda) \neq 0$ ist. Seien dies die Zahlen $\Lambda_1, \Lambda_2, \Lambda_3, \ldots$ Die entsprechenden $a(\lambda)$ werden durch $A_1, A_2, A_3, \ldots$ bezeichnet.

Nach der Besellschen Ungleichung konvergiert die Reihe $\sum |A_n|^2$. Also kann man für jedes $\varepsilon > 0$ ein solches $N = N(\varepsilon)$ angeben, dass $|A_n| < \varepsilon$, wenn $n > N$.

Betrachten wir die Funktion

$$R_N(x) = f(x) - \sum_{n=1}^{N} A_n e^{i\Lambda_n x}.$$

Offenbar ist

$$\sup_{-\infty < \lambda < \infty} \lim_{T \to \infty} \frac{1}{2T} \left| \int_{-T}^{T} R_N(x) e^{-i\lambda x} dx \right| < \varepsilon,$$

w. z. b. w.

Somit sehen wir, dass die Funktionen $f(x)$, deren Mittelwerte $a(\lambda)$ für alle reellen λ existieren, als das Resultat einer gewissen Abschliessung der Menge aller endlichen trigonometrischen Summen betrachtet werden können.

Also kann man die notwendige und hinreichende Bedingung in der Terminologie der Fastperiodizität ausdrücken.

Satz II. *Damit für alle reellen* λ *die Mittelwerte* $a(\lambda)$ *existieren, ist es notwendig und hinreichend, dass für jedes* $\varepsilon > 0$ *eine solche fastperiodische Zahlenmenge* (2) $\{\tau_n\}$ *existiere, dass*

$$\sup_{-\infty<\lambda<\infty} \overline{\lim_{T\to\infty}} \frac{1}{2T} \left| \int_{-T}^{T} \left\{ \overline{\lim} \sum_{j=-n}^{n} \frac{1}{c} \int_{x}^{x+c} [f(y+\tau_j) - f(y)]\, dy \right\} e^{-i\lambda x}\, dx \right| < \varepsilon$$

für jedes $c > 0$ ausfällt.*

Noch einfacher ist die notwendige und hinreichende Bedingung für die gleichmässige Existenz der Mittelwerte.

Satz III. *Damit für die Funktion* $f(x)$ *die Mittelwerte* $a(\lambda)$ *für alle reellen* λ *gleichmässig existieren, ist es notwendig und hinreichend, dass für jedes* $\varepsilon > 0$ *eine solche relativ-dichte Zahlenmenge* $\{\tau\} = \{\tau(\varepsilon)\}$ *und eine solche Zahl* $L = L(\varepsilon)$ *existieren, dass für jedes* τ

$$\sup_{\lambda} \sup_{x} \frac{1}{L} \left| \int_{x}^{x+L} [f(x+\tau) - f(x)]\, e^{-i\lambda x}\, dx \right| < \varepsilon.$$

2. Jetzt benutzen wir den folgenden Satz von S. Banach (3):

Es sei E *die Menge aller reellen beschränkten Funktionen im Intervalle* $[0, \infty]$.

Jeder Funktion $f(x) \subset E$ *kann man eine Zahl* $\operatorname{Lim}_{x\to\infty} f(x)$ *so zuordnen, dass die folgenden Beziehungen gelten* [$f(x)$ *und* $g(x)$ *sind Funktionen aus* E; a, b *und* $x_0 \geqslant 0$ *— reelle Zahlen*]:

1) $\operatorname{Lim}_{x\to\infty} [af(x) + bg(x)] = a \operatorname{Lim}_{x\to\infty} f(x) + b \operatorname{Lim}_{x\to\infty} g(x)$;

2) $\operatorname{Lim}_{x\to\infty} f(x) \geqslant 0$, für $f(x) \geqslant 0$;

3) $\operatorname{Lim}_{x\to\infty} f(x+x_0) = \operatorname{Lim}_{x\to\infty} f(x)$;

4) $\operatorname{Lim}_{x\to\infty} 1 = 1$;

5) $\underline{\lim}_{x\to\infty} f(x) \leqslant \operatorname{Lim}_{x\to\infty} f(x) \leqslant \overline{\lim}_{x\to\infty} f(x)$.

Aus 5) folgt, dass wenn $\lim_{x\to\infty} f(x)$ existiert**, so stimmt er mit $\operatorname{Lim}_{x\to\infty} f(x)$ überein.

Nun betrachten wir die Menge aller messbaren reellen Funktionen $f(x)$ $(-\infty < x < \infty)$, für welche

$$\overline{\lim_{T\to\infty}} \frac{1}{2T} \int_{-T}^{T} [f(x)]^2\, dx < \infty \qquad (4)$$

ist.

Wir wollen die Funktion

$$F(T) = \frac{1}{2T} \int_{-T}^{T} f(x)\, dx \qquad (T > 0)$$

einführen. Nach (4) ist $F(T) \subset E$.

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* Wir setzen voraus, ohne dadurch die Allgemeinheit zu beschränken, dass $f(x)$ eine reelle Funktion ist.

** Das Zeichen Lim bedeutet irgendeinen verallgemeinerten Limes, während das Zeichen lim für die Bezeichnung von Limes im gewöhnlichen Sinne vorbehalten ist.

Unter dem Mittelwert $M\{f(x)\}$ einer Funktion $f(x)$ werden wir $\operatorname{Lim}_{T\to\infty} F(T)$ verstehen. Es ist klar, dass der in dieser Weise bestimmte Mittelwert den folgenden Beziehungen genügt:

1) $M\{af(x)+bg(x)\}=aM\{f(x)\}+bM\{g(x)\}$ [a, b — reelle Zahlen; $f(x)$, $g(x)$ genügen (4)];

2) $M\{f(x)\}\geqslant 0$, wenn $f(x)\geqslant 0$;

3) $M\{f(x+x_0)\}=M\{f(x)\}$ (x_0—reell);

4) $M\{1\}=1$;

5) $\underline{\lim}_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} f(x)\,dx \leqslant M\{f(x)\} \leqslant \overline{\lim}_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} f(x)\,dx.$

Aus 5) folgt, dass, wenn $\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} f(x)\,dx$ existiert, er gleich $M\{f(x)\}$ ist.

In gewöhnlicher Weise kann man die Schwarzsche Ungleichung

$$M\{f(x)g(x)\}\leqslant\sqrt{M\{[f(x)]^2\}\cdot M\{[g(x)]^2\}}$$

herleiten. Setzen wir

$$a(\lambda)=M\{f(x)\cos\lambda x\},\quad b(\lambda)=M\{f(x)\sin\lambda x\}.$$

Wenn $\lambda_1, \lambda_2, \ldots \lambda_N$ beliebige voneinander verschiedene reelle Zahlen sind, so gelangt man auch, wie in der Theorie der fastperiodischen Funktionen von H. Bohr, zur Besselschen Ungleichung

$$\sum_{n=1}^{N}\{[a(\lambda_n)]^2+[b(\lambda_n)]^2\}\leqslant M\{[f(x)]^2\}.$$

Daraus folgt, dass es nur eine abzählbare λ-Menge gibt, für welche $a(\lambda)\neq 0$, $b(\lambda)\neq 0$ ist.

Bezeichnen wir diese letzten Werte (in beliebiger Ordnung) mit $0=\Lambda_0, \Lambda_1, \Lambda_2 \ldots$ Die entsprechenden $a(\lambda)$ und $b(\lambda)$ bezeichnen wir mit $A_0, A_1, A_2, \ldots$; $B_1, B_2, \ldots$

Somit kann man einer jeden reellen messbaren Funktion $f(x)$, die den Bedingungen (4) genügt, eine Fouriersche Reihe

$$f(x)\sim\sum(A_n\cos\Lambda_n x+B_n\sin\Lambda_n x)$$

zuordnen.

Für diese Fouriersche Reihe gilt der Eindeutigkeitssatz in der Form, in der er in [1] formuliert ist.

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ASTRONOMIE

SUR LE RÔLE DE LA MATIÈRE GALACTIQUE DANS LE PHÉNOMÈNE DE LA LUMIÈRE ZODIACALE

Par B. FESSENKOFF, de l'Académie

La statistique des météores observés pendant la nuit et les déterminations de leur vitesses dans les couches supérieures de l'atmosphère montrent que la plupart de ces corpuscules sont étrangers à notre système planétaire. D'ailleurs le percentage des météores sensiblement hyperboliques avec les vitesses à quelques centaines kilomètres devient d'autant plus considérable que leur éclat apparent diminue. Ainsi on peut s'attendre que l'ensemble des météorites dans l'espace planétaire appartient à quelques courants pas trop nombreux qui traversent notre système en venant de quelques endroits galactiques, par exemple, de celui occupé par les nebuleuses obscures dans la constellation de Taurus.

Considérons le problème suivant. Les particules du courant se meuvent suivant les lignes droites et parallèles avec la vitesse v_0. L'attraction du Soleil déformant les trajectoires produit les inégalités dans la distribution de la matière dans l'espace. Il faut calculer la densité dans n'importe quel point de l'espace et évaluer ensuite la luminosité apparente du ciel produite par ce courant météorique pour un observateur terrestre.

Supposons que le Soleil est dans l'origine des coordonnées, l'axe x étant orienté suivant la direction du courant. Un point matériel avec les coordonnées initiales $x_0 \to -\infty$, y_0 et la vitesse v_0 se meut suivant la trajectoire hyperbolique en coordonnées polaires r, ϑ

$$r = \frac{-a \operatorname{tg}^2 \psi}{1 + \cos\vartheta + \operatorname{tg}\psi \sin\vartheta},$$

où $a = -\frac{k^2}{v_0^2}$ est le demi-axe réel de l'hyperbole, l'angle ψ fixant son orientation. Comme on sait, ψ est lié aves l'excentricité par $e = \sec\psi$.

L'intégral des aires fournit en outre la relation: $-a \operatorname{tg}\psi = y_0$.

Par conséquent, les paramètres y_0, v_0 étant donnés, la trajectoire de la particule est déterminée complètement. La direction de la normale à la trajectoire est donnée par l'expression

$$\cos(n, r) = \left[1 - r^2 \frac{\sec^2\psi}{a(e^2-1)} \sin^2(\psi - \vartheta)\right]^{-\frac{1}{2}}.$$

Considérons maintenant à une grande distance du Soleil un élément rectangulaire $\delta x_0 \delta y_0$ qui est rempli par les particules gravitantes. En s'approchant du Soleil, cet élément se déforme. Désignons par B le rapport de son aire dans un point r, ϑ à l'aire initiale $\delta x_0 \delta y_0$. On trouve facilement

$$B=\frac{\partial r\, v\cos(n,r)}{\partial y_0\, v_0}=\frac{ar^2}{y_0^3}\left[\frac{y_0}{a}\sin\vartheta-2(1+\cos\vartheta)\right]\frac{\sqrt{1-\frac{2a}{r}}}{\sqrt{1+\frac{r^2}{y_0^4}(y_0\cos\vartheta+a\sin\vartheta)^2}}$$

Dans l'espace à trois dimensions la densité de la matière dans l'unité de volume est

$$D=D_1+D_2=\frac{1}{y}\left(\frac{y_0}{B}+\frac{y_0'}{B'}\right),$$

B, y_0, B', y_0' étant les valeurs de B, y_0 dans les points symétriques par rapport à l'axe de x.

Pour r très petit, ϑ étant arbitraire, la fonction $B(r,\vartheta)$ est 2; aussi $B=2$ pour ϑ très petit, r étant arbitraire. En général $1\leqslant B\leqslant 2$ et $2\leqslant B<\infty$ suivant que y_0 ou $y\gtrless 0$. La densité D étant connue, on calcule la brillance apparente du courant météorique observé de la Terre d'après la formule

$$J\,d\omega=\mu L\,d\omega\int_0^\infty\frac{D\,f(\alpha)\,dR}{\xi^2},$$

$Ld\omega$ étant la quantité de lumière du Soleil dans l'angle solide $d\omega$, $\mu f(\alpha)$ —le pouvoir diffusant de la matière, α — l'angle de phase, ξ, R — les distances de l'élément considéré du Soleil et de la Terre. Pour le plan de l'ecliptique on a:

$$J\,d\omega=\frac{L\mu d\omega}{R_0\sin l}\int_0^{\pi-l}Df(\alpha)\,d\alpha,$$

où l est la distance angulaire du Soleil de la région observée du ciel, R_0 — le rayon de l'orbite terrestre.

Supposons maintenant pour fixer les idées que $a=-0.1$. On obtient pour D les valeures suivantes:

ϑ \ r	0.2	0.4	0.6	0.8	1.0	1.5
2.9°	20.0	14.1	11.5	10.0	8.9	7.3
15°	3.99	2.79	2.34	2.08	1.90	1.63
30°	2.16	1.71	1.44	1.32	1.22	
60°	1.35	1.16	1.09	1.06	1.04	
90°	1.16	1.10	1.04	1.02	1.02	
120°	1.09	1.03	1.03	1.02	1.01	
177.1°	1.02	1.00	1.00	1.00	1.00	

En évaluant J pour la position de la Terre, correspondant à $\vartheta=60°$, j'ai obtenu en unité provisoire pour les branches de la lumière zodiacale

	$l=30°$	60°	90°
La branche de l'Ouest . .	8.5	3.9	2.7
La branche de l'Est . . .	4.4	1.8	1.2

Les deux branches sont presque identiques quant à la distribution de la brillance, mais tout à fait différentes quant à la brillance absolue. Donc, il est intéressant d'obtenir les observations photométriques sur la lumière zodiacale rapportant à deux branches de ce phénomène simultanément. Ces observations peuvent reveler les variations périodiques suivant la longitude de la Terre.

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HYDRAULICS

ON THE CALCULATION OF CHAMBERS USED FOR REDUCING PRESSURE OSCILLATIONS IN THE CASE OF UNSTEADY FLOW IN PIPES

By I. TCHARNYI

(Communicated by S. A. Tschaplygin, Member of the Academy, 17. III. 1938)

1. For reducing the surge in pipes in the case of unsteady fluid motion, chambers of large capacity, connected to the pipe-line, are applied. For instance, in reciprocating pumps such a chamber is the air chamber, in discharge pipe-lines of hydro-electrostations it is the surge tank or the stand pipe.

As a rule, technical calculations of such arrangements are made under the assumption that the fluid is incompressible. This leads to errors and disagreements with experiments. In this paper a general method of calculating chambers for reducing pressure oscillations is given, which takes into account the viscosity and the compressibility of the fluid.

Our problem can be formulated as follows: to the one end of the pipe-line an aggregate is attached which allows to change the flow in the pipe as a function of time, according to a certain law; such an aggregate is, for example, a reciprocating pump, a turbine, a valve etc. The aggregate is separated from the pipe-line by a chamber which serves for reducing pressure oscillations. The other end of the pipe-line is open, and a constant pressure which is assumed to be zero is there maintained.

We have to determine the capacity of the chamber for which the pressure oscillations will not exceed some allowable limits, or to find the pressure oscillations when the capacity of the chamber is known.

2. The general solution of this problem is given in our paper [1]; it is shown in [1] that this problem is reduced to the integration of the equation:

$$\frac{\partial^2 v}{dt^2} + 2a \frac{\partial v}{\partial t} = c^2 \frac{\partial^2 v}{\partial x^2} \tag{1}$$

under the boundary conditions:

$$\frac{\partial v}{\partial x} = 0 \qquad \text{for } x = 0, \tag{2}$$

at the open end of the pipe-line, and

$$v + h \frac{\partial v}{\partial x} = f(t) \qquad \text{for } x = l, \tag{3}$$

at the aggregate.

The pressure in the chamber is determined by the equation

$$\frac{p}{\rho} = \frac{p_1}{\rho} - \int_0^l \left(\frac{\partial v}{\partial t} + 2av \right) dx$$

which after some calculations carried out in [1], gives

$$\frac{p}{\rho} = \frac{p_1}{\rho} + \frac{p_{\text{forced}}}{\rho} + \frac{p_{\text{free}}}{\rho}, \tag{4}$$

where ρ is the density of the fluid, p_1 is the pressure in the chamber when the flow is stationary;

$$\frac{p_{\text{forced}}}{\rho} = -2\pi i \cdot c \cdot \sum_k \frac{\Phi(\omega_k)}{\lambda'(\omega_k)} \cdot Z(\omega_k) \cdot e^{i\omega_k t} \tag{5}$$

are the forced oscillations,

$$\frac{p_{\text{free}}}{\rho} = -2\pi \cdot \frac{c^2}{l} \cdot e^{-at} \times$$

$$\times \sum_{s=1}^{\infty} \frac{F(ia+\xi_s) \cdot e^{i\xi_s t} + F(ia-\xi_s) \cdot e^{-i\xi_s t} - \frac{ia}{\xi_s} \left[F(ia+\xi_s) \cdot e^{i\xi_s t} - F(ia-\xi_s) \cdot e^{-i\xi_s t} \right]}{\beta^2 \varphi_s^2 + \beta + 1} \tag{6}$$

are the free oscillations.

In formulae (1) — (6) the following denotations are used:

$$F(\omega) = \frac{1}{2\pi} \int_0^{\infty} f(\alpha) \cdot e^{-i\omega\alpha} \cdot d\alpha = \frac{\Phi(\omega)}{\lambda(\omega)}, \tag{a}$$

where the right-hand side is, in practically interesting cases, a rational fraction, the denominator of which $\lambda(\omega)$ has simple roots ω_k;

$$f(t) = \frac{Q(t)}{f} \tag{b}$$

where $Q(t)$ is the given flow, produced or regulated by the aggregate, measured from the stationary flow which existed at the time $t = 0$; f is the cross-section of the pipe-line; $f(t)$ has to satisfy the conditions of Dirichlet: $f(t) = 0$ when $t < 0$; $Q(t)$ is positive when it is directed from the chamber to the aggregate and negative when its direction is opposite.

$$Z(\omega) = \frac{i\omega + 2a}{\sqrt{\omega^2 - i \cdot 2a\omega} \cdot \left(\operatorname{ctg} \frac{\sqrt{\omega^2 - i \cdot 2a\omega}}{c} l - \frac{\sqrt{\omega^2 - i \cdot 2a\omega}}{c} h \right)}, \tag{c}$$

l is the length of the pipe-line,

$$c = \sqrt{\frac{K}{\rho}} \tag{d}$$

is the sound velocity in the fluid flowing in a pipe-line with elastic walls, K is the compressibility coefficient of the fluid in a pipe-line with elastic walls, which takes into account the combined effect of compressibility of the fluid and the expansion of the pipe-line.

$2a$ is a parameter answering for the viscous friction between the fluid and the walls of the pipe; ν is the coefficient of kinematic viscosity, d is the diameter of the pipe;

$$2a = \frac{32\nu}{d^2} \tag{e}$$

for the laminar flow and

$$2a = \frac{\lambda v_m}{2d} \tag{f}$$

for the turbulent flow; λ is the coefficient of resistance in D'Arcy-Weissbach's formula $\left(\lambda \frac{l}{d} \frac{v_m}{2g}\right)$ for the loss of pressure-head due to viscous friction, v_m is the mean (in time) velocity of the fluid in the pipe.

The formula (e) gives an exact account of the effect of viscosity, (f) is an approximate formula which gives, however, results sufficiently accurate for purpose of practice.

h is a coefficient, depending on the capacity of the chamber.

For the air chamber with isothermical regime and small pressure oscillations:

$$h = \frac{KV_0}{p_0 f} \tag{g}$$

where V_0 is the mean volume of the air in the air chamber and p_0 — the mean absolute pressure in the air chamber.

For a (prismatic) surge tank

$$h = \frac{F}{f} \cdot \frac{K}{\gamma} \tag{h}$$

where F is the cross-section of the surge tank, $\gamma = \rho g$ and $g = 9.81 \frac{\text{m}}{\text{sec}^2}$ is the acceleration of gravity.

$$\beta = \frac{h}{l}. \tag{i}$$

φ_s is the s-th root of the transcendental equation

$$\operatorname{ctg} \varphi_s - \beta \varphi_s = 0, \tag{k}$$

$$\xi_s = \sqrt{\varphi_s^2 \frac{c^2}{l^2} - a^2}. \tag{l}$$

In dimensional expression we have $[a] = T^{-1}$, $[h] = L$. It is clear that formulae (5) and (6) remain valid also in the case $h = 0$, i. e. when the chamber is absent.

3. In practice the most interesting cases are the impulsive and the periodical changes of the flow, which are considered below.

I. Impulsive change of the flow (hammer):

$$\left.\begin{aligned} f(t) &= A \quad (t > 0), \\ f(t) &= 0 \quad (t < 0). \end{aligned}\right\} \tag{7}$$

Here the free oscillations determined by (6) are the most interesting ones. It is shown in (1) that from (6) we obtain

$$\left.\begin{aligned} \frac{p_{\text{free}}}{\rho} &= -2Ac \cdot e^{-at} \cdot \sum_{s=1}^{\infty} \frac{\sin(\xi_s t - \theta_s) - \frac{a}{\xi_s} \cos(\xi_s t - \theta_s)}{\varphi_s (\beta^2 \varphi_s^2 + \beta + 1)}, \\ \operatorname{tg} \theta_s &= \frac{a}{\xi_s}, \end{aligned}\right\} \tag{8}$$

or

$$\frac{p_{\text{free}}}{\rho} = -2Ac \cdot e^{-at} \cdot \sum_{s=1}^{\infty} \frac{\sin(\xi_s t - 2\theta_s)}{\varphi_s (\beta^2 \varphi_s^2 + \beta + 1) \cos \theta_s}. \tag{8,1}$$

The series (8,1) is obviously convergent and may be used in technical calculations. As a rule β is large, $\beta > 10$. In this case, (8) may be simplified: when β is large, φ_1 is small and from (k) we obtain approximately $\operatorname{ctg} \varphi_1 \approx \frac{1}{\varphi_1}$:

$$\varphi_1 \approx \frac{1}{\sqrt{\beta}}, \tag{9}$$

$$\varphi_s \approx (s-1)\pi \qquad (s > 1). \tag{10}$$

In (8) or (8,1) we may then take only the first term since it can be shown that the sum of amplitudes of higher harmonics is less than 1% of the amplitude of the first harmonic, and from (8,1) it follows:

$$\frac{p_{\text{free}}}{\rho} \approx -2Ac \cdot e^{-at} \cdot \frac{\sin(\xi_1 t - 2\theta_1)}{\left(2\sqrt{\beta} + \frac{1}{\sqrt{\beta}}\right)\cos\theta_1}, \tag{11}$$

but a being usually small, $\cos\theta_1 \approx 1$ and $\frac{1}{\sqrt{\beta}}$ is small in comparison with $2\sqrt{\beta}$, so that

$$\frac{p_{\text{free}}}{\rho} \approx -\frac{Ac}{\sqrt{\beta}} \cdot e^{-at} \sin(\xi_1 t - 2\theta_1) \tag{11,1}$$

where, by (l), (8), (9),

$$\left.\begin{aligned} \xi_1 &= \sqrt{\frac{c^2}{\beta l^2} - a}, \\ \operatorname{tg}\theta_1 &= \frac{a}{\xi_1}. \end{aligned}\right\} \tag{12}$$

From (11,1) it is clear that an impulsive change of velocity A produces in the chamber a damped oscillation of pressure with the period $T = \frac{2\pi}{\xi_1}$ and the initial amplitude

$$\left|\frac{p_{\text{free}}}{\rho}\right| \approx \frac{Ac}{\sqrt{\beta}}.$$

(We neglect the damping, since a is small and ξ_1 usually very large.)

For the surge tank we have according to (h)

$$\beta = \frac{F}{f} \cdot \frac{K}{\gamma l} = \frac{F}{f} \cdot \frac{c^2}{gl}, \tag{13}$$

$g = 9.81 \frac{\text{m}}{\text{sec}^2}$ and from (11,1) we obtain a formula for the calculation of the maximal amplitude of the oscillating level in the surge tank:

$$|H| = \left|\frac{p_{\text{free}}}{\gamma}\right| \approx A \cdot \sqrt{\frac{f}{F}} \cdot \sqrt{\frac{l}{g}}. \tag{14}$$

For the air chamber we have according to (g)

$$\beta = \frac{KV_0}{p_0 f l} = \frac{\rho V_0}{p_0 f l} c^2$$

and from (11,1)

$$\left.\begin{aligned} \left|\frac{p_{\text{free}}}{\gamma}\right| &\approx A \cdot \sqrt{\frac{fl}{V_0}} \cdot \sqrt{\frac{h_0}{g}}, \\ h_0 &= \frac{p_0}{\gamma}. \end{aligned}\right\} \tag{15}$$

II. Periodical change of the flow:

$$\left.\begin{aligned} f(t) &= Ae^{iqt} \qquad (t>0),\\ f(t) &= 0 \qquad\quad (t<0). \end{aligned}\right\} \tag{16}$$

The forced oscillations are easily found with the aid of Fourier series, but may be also determined from (5). From (a) we deduce

$$F(\omega)=\frac{1}{2\pi}\int_0^\infty A\cdot e^{-i(\omega-q)\alpha}\,d\alpha=\frac{A}{2\pi i}\cdot\frac{1}{\omega-q}=\frac{\Phi(\omega)}{\lambda(\omega)} \tag{17}$$

whence

$$\left.\begin{aligned} \Phi(\omega) &= \frac{A}{2\pi i},\\ \lambda(\omega) &= \omega - q,\\ \omega_1 &= q. \end{aligned}\right\} \tag{18}$$

From (c) it follows

$$Z(\omega_1)=Z(q)=\frac{iq+2a}{\sqrt{q^2-i\cdot 2aq}}\cdot\frac{1}{\operatorname{ctg}\frac{\sqrt{q^2-i\cdot 2aq}}{c}\,l-\frac{\sqrt{q^2-i\cdot 2aq}}{c}\,h}=$$

$$=|Z(q)|\cdot e^{-i\delta}, \tag{19}$$

and after some calculations,

$$|Z(q)|=\frac{r_1}{r_2}$$

where

$$\left.\begin{aligned} r_1 &= \sqrt[4]{\frac{q^2+4a^2}{q^2}};\\ r_2 &= \sqrt{\left(\frac{\sin 2\varphi}{\operatorname{ch}2\psi-\cos 2\varphi}-\beta\varphi\right)^2+\left(\frac{\operatorname{sh}2\psi}{\operatorname{ch}2\psi-\cos 2\varphi}+\beta\psi\right)^2};\\ \varphi &= \frac{l}{c}\sqrt{\frac{\sqrt{q^4+4a^2q^2}+q^2}{2}};\\ \psi &= \frac{l}{c}\sqrt{\frac{\sqrt{q^4+4a^2q^2}-q^2}{2}};\\ \delta &= \theta_3-\theta_2+\theta_1 \end{aligned}\right\} \tag{20}$$

where

$$\left.\begin{aligned} \operatorname{tg}\theta_1 &= \frac{\psi}{\varphi};\\ \operatorname{tg}\theta_2 &= \frac{\frac{\operatorname{sh}2\psi}{\operatorname{ch}2\psi-\cos 2\varphi}+\beta\psi}{\frac{\sin 2\varphi}{\operatorname{ch}2\psi-\cos 2\varphi}-\beta\varphi};\\ \operatorname{tg}\theta_3 &= \frac{q}{2a}. \end{aligned}\right\} \tag{21}$$

From (5), (18) and (19) we finally find

$$\frac{P_{\text{forced}}}{\rho}=-cA\,|Z(q)|\cdot e^{i(qt+\delta)}. \tag{22}$$

For the difference between the maximal and the minimal pressure we obtain from (22):

$$\left(\frac{\Delta p}{\rho}\right)_{\text{forced}} = \left(\frac{p_{\text{forced}}}{\rho}\right)_{\max} - \left(\frac{p_{\text{forced}}}{\rho}\right)_{\min} = 2Ac\,|Z(q)|,$$

or, since by (16) $2A = v_{\max} - v_{\min} = \Delta v$, where Δv is the difference between the maximal and the minimal velocity at the aggregate,

$$\left(\frac{\Delta p}{\rho}\right)_{\text{forced}} = c \cdot \Delta v \cdot |Z(q)|. \tag{22,1}$$

If $f(t)$ is a sum of periodical functions, for instance, if $f(t)$ is given by a Fourier series, the forced oscillations are determined by superposition of (22).

In most cases it is enough to consider the first harmonic.

In practice q is much greater than a; this allows to simplify (20):

$$\left.\begin{aligned} r_1 &\approx 1, \\ \varphi &\approx \frac{ql}{c}, \\ \psi &\approx \frac{al}{c}. \end{aligned}\right\} \tag{20,1}$$

The previous formulae may be applied to the investigation of various cases of the flow in pipe-lines with the chamber and without it.

The experimental data [the experiments of D. Z. Losinsky (3), Diederichs and Pomeroy (4), Berg (2)] confirm the theory developed in this paper. For small values of $\varphi \approx \frac{ql}{c}$ the above formulae reduce to Berg's formulae for incompressible fluids.

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PHYSIQUE MATHÉMATIQUE

SUR LES CENTRES ET LES FOYERS

Par I. KOUKLES

(Présenté par I. M. Vinogradow, de l'Académie, le 1. IV. 1938)

1. Le but de la présente Note est d'étudier les points singuliers du plan de phases pour l'équation

$$\frac{dy}{dx} = -\frac{F(x, y)}{y} \qquad (1)$$

(F étant une fonction analytique) dans le cas où ces points sont des centres ou bien des foyers. On sait que les méthodes connues [de Bendixon (1) et de Frommer (2)] pour distinguer un centre d'un foyer ne donnent pas des conditions suffisantes pour trouver un centre, et un foyer ne se trouve dans ces méthodes qu'après un grand nombre d'opérations et d'ailleurs ce nombre reste indéterminé. Il est si difficile de trouver les fonctions successives u_n par la méthode de Bendixon [il est facile à voir que pour l'équation (1) cette méthode coïncide avec la méthode des approximations successives de M. Picard] ou bien de trouver les quantités B_n par la méthode de Frommer que l'application de ces méthodes est pratiquement impossible même dans des cas très simples. H. Dulac, en suivant la méthode de Bendixon, a pu distinguer un centre d'un foyer (3) pour le cas d'une équation dont le second membre est une fraction ayant pour numérateur et pour dénominateur des polynômes du second degré. En 1934 M. Frommer a résolu le même problème en suivant sa propre méthode *. Il me semble qu'on ne trouve nulle part dans la littérature mathématique la solution du problème sur la distinction des centres et des foyers pour les cas plus compliqués.

Le but de la présente Note est d'indiquer une nouvelle méthode pour distinguer un centre d'un foyer; cette méthode tout en étant plus effective et plus commode pour les applications que celles de Bendixon et Frommer permet en certains cas d'établir l'existence d'un centre au moyen d'un nombre fini d'opérations. Cette méthode a pour base les théorèmes suivants que nous exposerons dans l'ordre de généralité croissante.

1) Désignons par A la partie de la fonction $F(x, y)$ impaire par rapport à y et par B la partie de cette fonction paire par rapport à x. Nous supposerons dans la suite que l'équation des tangentes possibles

* Récemment Moisseiev (4) a montré que les soi-disantes méthodes de Bendixon et de Frommer furent proposées pour la première fois par Liapounoff (5).

de Poincaré possède dans tous les cas des racines complexes ce qui correspond à l'hypothèse que l'origine des coordonnées est un centre ou bien un foyer.

Théorème 1. *Dans le domaine où la fonction* $\frac{A}{y}$ *sans être identiquement nulle conserve un signe constant ou bien la fonction* $\frac{B}{y}$ *sans être identiquement nulle conserve un signe constant, il n'existe aucune caractéristique fermée qui entoure l'origine*. Mais si* $A=0$, *l'origine et tous les autres points singuliers de l'équation* (1) *(sauf les cols) seront des centres. Si* $B=0$, *l'origine sera un centre tandis que les autres points singuliers peuvent être des foyers.*

Corrollaire. Dans le cas où $F(x,y)=f(x)+\varphi(y)$ la condition nécessaire et suffisante pour l'existence d'un centre est $A=0$. Dans le dernier cas l'équation n'a pour points singuliers que des centres ou des cols.

Exemple 1.

$$\frac{dy}{dx}=-\frac{x+ay^3+bx^2y^2+cx^2}{y};\quad \frac{A}{y}=ay^2.$$

Si $a\neq 0$ l'équation n'a aucune caractéristique fermée. Dans le cas contraire l'origine est un centre.

Exemple 2.

$$\frac{dy}{dx}=-\frac{x+ayx^3+by^2x^3+cx^4y+dx^5y^2+ex^5y^3}{y};$$

$$\frac{A}{y}=x^3(a+cx+ex^2y^2);\quad \frac{B}{y}=cx^4.$$

Si $c\neq 0$ il n'existe aucune caractéristique fermée. Dans le cas contraire l'origine est un centre.

Exemple 3.

$$\frac{dy}{dx}=-\frac{x+\sum_2^\infty \alpha_n x^n+\sum_1^\infty \beta_k x^k}{y};\quad \frac{A}{y}=\sum_1^\infty \beta_{2k+1}y^{2k}.$$

Si les β_{2k+1} ne sont pas tous nuls, l'équation n'a aucun point singulier du type d'un centre. Cependant ici on peut avoir des cycles limites dont le nombre peut être égal au nombre des racines positives de l'équation $\frac{A}{y}=0$, dont l'ordre de multiplicité est impaire.

Construisons maintenant deux fonctions $A^*(x,y)$ et $B^*(x,y)$ de la manière suivante: soit $u(x)$ et $v(y)$ deux fonctions quelconques vérifiant les conditions:

$$u(0)=u'(0)=0;\quad u''(0)>0 \tag{2}$$

et

$$v(0)=v'(0)=0;\quad v''(0)>0. \tag{3}$$

Représentons notre fonction $F(x,y)$ dans l'une des deux formes:

1) $F(x,y)=\Phi_0(u,y)u'+B^*(x,y)$

ou bien

2) $\frac{F(x,y)}{y}=\frac{\Phi_1(x,v)}{v'}+\frac{A^*(x,y)}{y}$,

* Sauf peut-être quelques cycles limites dont les équations sont $AB=0$.

où Φ et Φ_1 sont des fonctions analytiques. On peut démontrer que ou bien il est possible de choisir la fonction Φ de telle manière que $B^\star$ soit nul, ou bien il existe un polynôme Φ tel que $B^\star$ conserve son signe pour les petites valeurs de x et pour y fixés. De même, ou bien il existe une fonction Φ_1 telle que $A^\star$ est nul, ou bien il existe un polynôme Φ_1 tel que la fonction $\frac{A^*}{y}$ conserve son signe pour les petites valeurs de y et pour x fixés; ces deux propriétés ont lieu pour toutes les fonctions analytiques $u(x)$ et $v(y)$ vérifiant les conditions (2) et (3)*.

Théorème 2. *Dans le domaine, où la fonction* $\frac{A^*}{y}$ *ou bien la fonction* $\frac{B^*}{y}$ *conserve son signe, il n'existe aucune caractéristique fermée entourant l'origine* **. *Si* $A^\star = 0$, *l'origine et les autres points singuliers (excepté les cols) sont des centres. Si* $B^* = 0$, *l'origine est un centre tandis que les autres points singuliers peuvent être des cols.*

Il est facile à voir que le théorème 1 est un cas particulier du théorème 2, car en posant $u = \frac{1}{2}x^2$ et $v = \frac{1}{2}y^2$ on voit que les fonctions $A^\star$ et $B^\star$ dégénèrent et deviennent respectivement égales aux fonctions A et B du théorème 1.

Corrollaire. Pour l'équation

$$\frac{dy}{dx} = -\frac{f(x)\varphi(y) + f_1(x)\varphi_1(y)}{y} \tag{2'}$$

la condition nécessaire et suffisante pour l'existence d'un centre à l'origine est la condition $A^\star B^\star = 0$.

Remarque. Si $f(x)$ contient x en premier degré on peut poser $u = \int_0^x f(\alpha)\,d\alpha$. Dans le cas contraire on peut poser $u = \left[\int_0^x f(\alpha)\,d\alpha\right]^{\frac{1}{n+1}}$, où l'on suppose que le terme de $f(x)$ dont le degré est le plus petit est de degré $2n+1$ tandis que le terme de $f_1(x)$ dont le degré est le plus petit est de degré $\geqslant 2n+1$. Dans ce cas $\varphi(y)$ contient nécessairement un terme constant ce qui permet de poser $v = \int_0^y \frac{y\,dy}{\varphi(y)}$.

Exemple 4.

$$\frac{dy}{dx} = -\frac{f_n(x)(y-\alpha)(y+\beta) + \varphi_q(y) f_n(x) F_p(x)}{y},$$

où α et β sont des nombres positifs rationnels, f_n, φ_q et F_p sont des polynômes et f_n contient x en premier degré.

La première condition ($B^\star = 0$) est vérifiée dans le cas et dans ce cas seulement où

$$F_p(x) = \Phi\left[P_{k+1}(x)\right] \tag{4}$$

et

$$P_{k+1}(x) = \int_0^x f_n(\alpha)\,d\alpha.$$

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* Les polynômes précédemment indiqués sont déterminés d'une manière unique si l'on exige qu'ils possèdent le moindre degré parmi tous les polynômes possibles.

** Sauf peut-être quelques cycles limites dont les équations sont: $A^*B^* = 0$.

La condition (4) est vérifiée, ce qu'il est facile à voir, en raisonnant de la manière suivante: la condition (4) est vérifiée dans le cas et dans ce cas seulement, où les deux polynômes F_p et P_{k+1} sont des polynômes d'un certain polynôme $q_e(x)$ dont le degré est le diviseur commun (différent de l'unité) des degrés des deux polynômes donnés [en particulier la fonction $q_e(x)$ peut coïncider avec l'un des polynômes donnés]. Ainsi, par exemple, si P_{k+1} est un polynôme du 5-ième degré ou bien une fonction analytique de ce polynôme, $F_p(x)$, étant un polynôme, ne peut avoir pour degré qu'un multiple de 5.

Si la première condition n'est pas vérifiée, la fonction $\varphi_k(y)$ doit vérifier la seconde condition ($A^* = 0$), donc nous devons avoir:

$$\varphi(y) = (y-\alpha)(y+\beta)\ \Phi\left[(y+\alpha)^{\frac{\alpha}{\alpha+\beta}}(y+\beta)^{\frac{\beta}{\alpha+\beta}}\right],$$

où Φ est une fonction analytique. Comme α et β sont rationnels et positifs, en posant

$$\frac{\alpha}{\alpha+\beta} = \frac{k}{n};\quad \frac{\beta}{\alpha+\beta} = \frac{n-k}{n} \tag{5}$$

(où k et n sont des entiers positifs et $n > k$) nous aurons:

$$\varphi(y) = \Phi_1[(y-\alpha)^k (y+\beta)^{n-k}](y-\alpha)(y+\beta).$$

Remarquons que les plus petites valeurs de k et n sont déterminées par la condition que la fraction $\frac{k}{n}$ déterminée par l'équation (5) soit irréductible.

Considérons maintenant l'équation du type le plus général

$$\frac{dy}{dx} = \Omega(x, y), \tag{6}$$

où Ω est une fonction quelconque telle que l'origine des coordonnées est pour l'équation (6) un foyer ou bien un centre.

Théorème 3. *Pour que l'équation* (6) *possède à l'origine des coordonnées un point singulier du type de centre il est nécessaire et suffisant que la condition suivante soit vérifiée*:

$$\Omega(x, y) = \frac{\Phi(u, v)\, u'_x - v'_x}{v'_y - \Phi(u, v) u'_y}, \tag{6'}$$

où les fonctions $u(x, y)$ et $v(x, y)$ sont assujetties aux restrictions: ou bien a) $u(0, y) = 0$; $\frac{\partial u}{\partial x}(0, y) = 0$; $\frac{\partial^2 u}{\partial x^2}(0, y) > 0$ et d'ailleurs $v(x, 0) = 0$ et $\frac{\partial v}{\partial y}(x, 0) > 0$, ou bien b) $u(0, y) = 0$, $\frac{\partial u}{\partial x}(0, y) > 0$ et d'ailleurs $v(x, 0) = 0$; $\frac{\partial v}{\partial y}(x, 0) = 0$; $\frac{\partial^2 v}{\partial y^2}(x, 0) > 0$.

La fonction Φ peut être arbitraire uniforme. Remarquons d'ailleurs que si la fonction Φ est telle que l'équation $\frac{dv}{du} = \Phi(u, v)$ admet une famille de caractéristiques qui coupent l'axe OY au-dessus et au-dessous de l'origine [quand la condition a) est vérifiée] ou bien l'axe OX à droite et à gauche de l'origine [quand la condition b) est vérifiée], l'équation (6) aura nécessairement un centre à l'origine. Mais si la fonction uniforme Φ ne jouit pas de cette propriété, l'origine des coordonnées peut présenter pour l'équation (6) un point singulier d'une autre nature ou bien un point ordinaire (mais en tout cas il ne sera pas un foyer).

Il est facile à voir qu'en particulier, si nous posons $u = u(x)$; $v = y$ ou bien $u = x$; $v = v(y)$, on obtient les conditions suffisantes pour un centre énoncées dans le théorème 2.

Il est démontré que si la fonction $y\Omega(x, y)$ est analytique, alors ou bien il existe une fonction analytique $\Phi(u, v)$ on bien l'origine n'est pas un centre.

Développons $F(x, y)$ [le numérateur du second membre de l'équation (1)] en une série suivant les puissances de y:

$$F(x, y) = \omega_0(y) + x\omega_1(y) + \ldots + x^n\omega_n(y) + \ldots$$

En appliquant la condition b) du théorème 3 nous obtenons l'expression la plus générale pour les fonctions analytiques ω_n vérifiant la condition du centre:

$$\begin{aligned} -\omega_n(y) = \frac{1}{\psi_0}\Big[& \psi_1\omega_{n-1} + \psi_2\omega_{n-2} + \ldots + \psi_n\omega_0 - (n+1)\varphi_{n+1} + \\ & + f_n(\varphi_0) + \varphi_1 f'_{n-1}(\varphi_0) + \varphi_2 f'_{n-2}(\varphi_0) + \frac{\varphi_1^2}{2!} f''_{n-1}(\varphi_0) + \varphi_3 f'_{n-3}(\varphi_0) + \\ & + \frac{2\varphi_1\varphi_2}{2!} f''_{n-2}(\varphi_0) + \frac{\varphi_1^3}{3!} f'''_{n-1}(\varphi_0) + \ldots + \frac{\varphi_1^n}{n!} f_0^{(n)}(\varphi_0) + \ldots \Big], \end{aligned} \tag{7}$$

où la fonction

$$\varphi_0(y) = \frac{1}{2}y^2 + \lambda_3 y^3 + \lambda_5 y^5 + \ldots + \lambda_{2k+1} y^{2k+1} + \ldots$$

les fonctions φ_1, φ_2, ... sont des fonctions analytiques arbitraires de y, ne contenant que des termes de degré supérieur ou égal à 2, les fonctions $\psi_n(y) = \frac{1}{y}\varphi'_n(y)$ et les fonctions f_n sont des fonctions analytiques arbitraires de φ_0. Remarquons d'ailleurs que dans le cas de l'existence du centre, quelles que soient les fonctions données ω_n, l'égalité (7) doit être vérifiée quelles que soient les fonctions données f_n (on peut même les supposer toutes nulles ou constantes). Quant aux coefficients λ_{2n+1} de la fonction φ_0, ils sont déterminés d'une manière unique quand les coefficients des fonctions ω_n sont donnés et les coefficients des fonctions f_n sont choisis; ensuite les coefficients λ_{2k+1} étant connus, on détermine d'une manière unique les fonctions φ_1, φ_2 et ainsi de suite. En déterminant la fonction φ_n de l'équation (7), nous obtenons ainsi le critère suivant pour distinguer un foyer d'un centre: chaque fonction $\varphi_n(y)$, définie par l'égalité

$$\begin{aligned} n\varphi_n(y) = {} & \psi_0\omega_{n-2} + \psi_1\omega_{n-2} + \ldots + \psi_{n-1}\omega_0 + f_n(\varphi_0) + \varphi_1 f_{n-1}(\varphi_0) + \\ & + \varphi_2 f'_{n-2}(\varphi_0) + \frac{\varphi_1^2}{2!} f''_{n-1}(\varphi_0) + \varphi_3 f'_{n-3}(\varphi_0) + 2\frac{\varphi_1\varphi_2}{2!} f''_{n-1}(\varphi_0) + \\ & + \frac{\varphi_1^3}{3!} f'''_{n-1}(\varphi_0) + \ldots + \frac{\varphi_1^n}{n!} f^{(n)}(\varphi_0), \end{aligned}$$

ne doit pas contenir des termes du premier degré en y, quelles que soient les fonctions f_n.

Exemple 5. Soit $F(x, y)$ un polynôme du second degré, c'est-à-dire $F(x, y) = x + ax^2 + bxy + cy^2$. En annulant les membres du premier degré de φ_1 et φ_2, on obtient la condition nécessaire pour un centre:

$$b(a + c) = 0.$$

Ainsi qu'il a été démontré par Frommer (2), cette condition est aussi suffisante.

Exemple 6. Soit $F(x, y)$ un polynôme du 3-ième degré, c'est-à-dire, $F(x, y) = x + ax^2 + bxy + cy^2 + dx^3 + ex^2y + fxy^2 + hy^3$. Pour distinguer un foyer d'un centre il faudra écrire 4 relations entre les coefficients en annulant successivement les membres du premier degré des fonctions φ_3, φ_5, φ_7 et φ_9 *. Après avoir fait les substitutions et simplifications nécessaires, on obtient les résultats suivants:

1) ou bien $a = c \neq 0$, $b \neq 0$, $e = h = 0$, $f = d = 0$;
2) ou bien $a = c = 0$, $e = h = 0$, f, d et b sont arbitraires;
3) ou bien $b = e = h = 0$, a, c, f et d sont arbitraires;
4) ou bien $c = 0$, $e = ab$, $h = d = f = 0$, a, b sont arbitraires.

On peut démontrer facilement que ces conditions sont suffisantes pour un centre. On peut arriver aux mêmes résultats en suivant la méthode de Frommer. Cependant il serait très difficile d'arriver au résultat précédemment indiqué en suivant la méthode de Frommer car les calculs sont très compliqués.

Considérons maintenant l'équation du type suivant:

$$\frac{dy}{dx} = -\frac{x + x^{2n}\omega_{2n}(y) + x^{2n+1}\omega_{2n+2}(y)}{y}, \tag{8}$$

où les $\omega_{2n}(y)$ et $\omega_{2n+2}(y)$ sont des polynômes ne contenant que des termes de degré impair.

Théorème 4. *La condition nécessaire et suffisante pour l'existence à l'origine d'un centre des caractéristiques de l'équation* (8) *est que toutes les fonctions* $\omega_{2n}(y)$ *et* $\omega_{2n+2}(y)$ *soient nulles, c'est-à-dire, que l'équation prenne la forme* $\frac{dy}{dx} = -\frac{x}{y}$.

Remarquons que pour démontrer ce théorème nous avons utilisé essentiellement la méthode qui résulte du théorème 3. Quant aux méthodes de Bendixon et Frommer, leur application à l'équation (8) conduit à des calculs très pénibles.

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* En annulant les fonctions $\psi_n(y_0)$ à indices pairs on n'obtient aucune nouvelle relation entre les coefficients de l'équation, mais on peut trouver les coefficients λ_{2n+1} de la fonction φ_0.

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PHYSIQUE

ÉTUDE DE LA FLUORESCENCE DANS UNE COUCHE D'ÉPAISSEUR COMPARABLE À LA LONGUEUR D'ONDE. II. LA POLARISATION DE LA FLUORESCENCE

Par F. S. BARYŠANSKAJA

(Présenté par L. I. Mandelstam, de l'Académie, le 15. III. 1938)

Dans la première communication (1), consacrée à ce problème, j'ai envisagé le phénomène de l'extinction de concentration de la fluorescence dans des couches minces. Les couches fluorescentes qu'on observe lors du phénomène de la réflexion totale à la limite du verre et de la solution fluorescente étant extrêmement minces, on peut étudier des solutions de très grandes concentrations, sans tenir compte des corrections qui seraient dues à l'absorption de la lumière, tant primaire que secondaire.

Cette même particularité du phénomène permet d'en user pour l'étude du problème de la polarisation de la fluorescence à de grandes concentrations. On connaît qu'une polarisation sensible de la fluorescence, dépendant fortement de la concentration (2), est observée dans les solutions de la fluorescéine dans la glycerine. Cependant, les données numériques se rapportant à de grandes concentrations ne peuvent être considérées comme exactes, car il n'y est pas tenu compte des corrections dues à la superposition des spectres d'absorption et de fluorescence. S. I. Wawilow(3) a récemment proposé une méthode indirecte pour introduire ces corrections; cette méthode s'applique, comme il suit de son raisonnement, à des cas limités par certaines suppositions spéciales. Les mesures effectuées par Wawilow, Chvostikov et Gluschkov (4) montrent apparemment que ces suppositions sont légitimes dans le cas étudié par eux. L'étude de la polarisation de la fluorescence dans des couches extrêmement minces permet évidemment de supprimer la nécessité d'introduire des corrections pour l'absorption; cette méthode peut par conséquent présenter de certains avantages pour l'étude du problème qui nous occupe.

§1. Méthode de mesure

La méthode employée pour obtenir une couche extrêmement mince était la même que dans la première communication (1). Mais pour rendre possible d'effectuer des observations non seulement de côté, mais aussi d'en haut, le prisme (de flint lourd) avait une forme convenablement choisie. La lumière provenant de la source allait en cône convergent dont l'axe, normal à la face latérale du prisme, formait un angle d'environ 15° avec la surface de

séparation. Lorsqu'on observait de côté, l'axe du cône de la lumière de fluorescence, normal à la seconde face, formait aussi un angle d'environ 15° avec la surface de séparation. La polarisation du faisceau primaire était effectuée à l'aide d'un polaroïde; le degré de polarisation était étudié par un polarimètre de Cornu. Pour exclure l'effet de la lumière diffusée gênante, on employait la méthode des filtres croisés. Une attention spéciale était en outre prêtée à la qualité de la polissure du prisme. Avant l'expérience, on lavait sa surface d'une faible solution d'acide fluorique, qu'on enlevait ensuite en rinçant le prisme d'abord d'eau distillée et en dernier lieu de la solution de fluorescéine qu'on allait étudier.

L'étude de la polarisation de la fluorescence à réflexion totale présente les particularités suivantes: il faut tenir compte, en premier lieu, de l'effet de polarisation possible quand la lumière passe d'un milieu à l'autre; en second lieu, de cette circonstance que le champ lumineux est très complexe dans ce cas.

On peut considérer le phénomène de réflexion totale comme une variation rapide de la direction du flux lumineux à travers la surface de séparation. Les calculs de Picht et Nöther[5] montrent que la distribution du champ dépend essentiellement de l'angle d'ouverture du faisceau incident, ainsi que de l'angle d'incidence et de l'indice de réfraction. Ces quantités ayant des valeurs normales, le changement de direction du flux lumineux a lieu sur une longueur de quelques dizaines d'ondes lumineuses. Donc, le flux lumineux pour ainsi dire serpente le long de la surface de séparation, de façon que le vecteur électrique peut changer rapidement de direction. Ne détaillant pas les différents modes possibles des directions de polarisation du rayon primaire, nous ne considérons que deux cas, présentant le plus d'intéret.

Premier cas. La direction du vecteur électrique est normale au plan d'incidence; la direction d'observation, du haut en bas, normalement à la surface de séparation. Dans ce cas la direction d'observation reste normale à la direction du vecteur électrique, quelle que soit la sinuosité du flux. Dans le cas de diffusion par de petites particules isotropes, la lumière serait totalement polarisée; la lumière de fluorescence doit être dépolarisée d'une façon dépendant de la concentration. La disposition exposée est donc commode pour l'étude du problème qui nous occupe—la dépolarisation de la lumière de fluorescence à grandes concentrations dans une couche extrêmement mince. On peut, pour comparer, ôter le prisme et observer la polarisation dans une couche d'épaisseur finie. Bien qu'alors la direction d'observation ne soit pas exactement normale à la direction du faisceau primaire, cette circonstance n'a pas néanmoins d'importance essentielle vu le choix donné de direction d'oscillations. En revanche, on n'a pas à introduire des corrections pour tenir compte de la polarisation résultant du passage d'un milieu à l'autre, lorsque l'on observe normalement à la surface de séparation et à la surface verre—air; ceci découle de la symétrie du problème.

Deuxième cas. La direction des oscillations est dans le plan d'incidence, la direction d'observation—de côté, normalement à la face latérale du prisme. Dans ce cas, la direction des oscillations est aussi presque normale à la direction d'observation (elles forment un angle d'environ 75°). Par conséquent, la dépolarisation de la lumière de fluorescence doit avoir sa valeur habituelle, corrigée par les formules de Fresnel pour le passage par la surface liquide—air. Lorsque l'on observe en réflexion totale, il faut tenir compte des sinuosités du flux lumineux, grâce auxquelles la direction des oscillations de la lumière primaire dans le second milieu varie. En conséquence, la valeur moyenne observée de la polarisation doit être approximativement nulle.

§2. Résultats des mesures

Premier cas. Observation d'en haut. A. Solution de fluorescéine dans l'eau. Dans ce cas, comme l'on sait, la polarisation de la fluorescence est presque nulle. Les expériences effectuées avec ces solutions servaient à contrôler le degré d'élimination de la lumière diffusée dans notre arrangement, car la lumière diffusée accuse une polarisation sensible. Les résultats donnés dans le tableau 1 montrent que la lumière diffusée était éliminée.

Tableau 1

Solution de fluorescéine dans l'eau: $C=10^{-3}$ g/cm³

Couche épaisse (sans prisme) . . . . . Polarisation = 0
Couche mince (avec prisme) . . . . . Polarisation = 0

B. Solution de fluorescéine dans la glycérine. Les résultats sond donnés dans le tableau 2.

Tableau 2

Concentration	Couche épaisse		Couche mince	
	Wawilow etc. (valeurs observées)	Auteur (valeurs observées)	Wawilow etc. (valeurs observées)	Auteur (valeurs observées)
$3\cdot10^{-4}$	22.5	20	26.5	26
10^{-3}	16.5	16	21.5	20
$3\cdot10^{-3}$	8.5	10	13.5	13
10^{-2}	1	1	.5	2.5

Les colonnes 2 et 4 renferment les données de Wawilow etc., trouvées d'après les courbes publiées.

La comparaison des résultats, calculés par la méthode de Wawilow, avec ceux, obtenus dans les expériences de l'auteur, montre une coïncidence entièrement satisfaisante.

Deuxième cas. Observation de côté. Il est difficile dans ce cas d'éviter entièrement les corrections, tenant compte de la polarisation introduite par le passage de la lumière par la surface de séparation sous de grands angles. C'est pourquoi nous avons trouvé préferable d'exclure ces corrections de la façon suivante.

On comparait la polarisation de la lumière provenant de notre dispositif à différentes orientations du polaroïde qui polarisait le faisceau primaire. Dans ces conditions toutes les corrections concernant le passage de la lumière par la surface de séparation restent identiques. Par conséquent, les résultats des comparaisons ne doivent dépendre que des processus primaires d'excitation de la fluorescence. Dans les conditions habituelles d'observation, la polarisation de la fluorescence est nulle quand le vecteur électrique coïncide avec la direction d'observation et diffère sensiblement du zéro quand le vecteur électrique est normal à la direction d'observation. Dans le cas considéré, il faut tenir compte dans ces deuxièmes observations de la sinuosité du flux lumineux, qui doit rendre la polarisation nulle, comme il a déjà été mentionné. L'expérience était effectuée de telle façon, que le degré de polarisation était mesuré à différentes positions du polaroïde. On a trouvé qu'il restait constant, ne dépendant ni de l'orientation du polaroïde ni de la concentration de la solution (de 10^{-4} à 10^{-2} g/cm³). Cette polarisation restante

(d'environ 10%) correspond par sa grandeur aux corrections prévues pour le passage de la lumière par la surface de séparation (12% environ) et doit donc être mise au compte de ces dernières. Par conséquent, la dépolarisation primaire ayant lieu lors de l'excitation de la fluorescence dans le cas, où le vecteur électrique du rayon incident est perpendiculaire à la direction d'observation, ne diffère pas du cas, où ce vecteur coïncide avec la direction d'observation, c'est-à-dire elle est approximativement nulle. Il faut voir dans ce fait la démonstration directe de la sinuosité du flux lumineux à la limite de deux milieux lors de la réflexion totale.

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PHYSICS

ON SOME PECULIARITIES OBSERVED IN THE LIQUEFACTION OF HELIUM

By I. L. ZELMANOV

(Communicated by A. F. Joffe, Member of the Academy, 1. IV. 1938)

The problem of the yield of liquid helium in the liquefaction process based on the utilization of Joule-Thomson effect has been repeatedly discussed(1). The $T—S$ diagram published by Keesom and Hanthoff(2) allows to solve unambiguously the problem of the liquefaction coefficient for any case whatsoever. However, the maximum of the liquefaction coefficient observed long ago by Kammerling-Onnes at a pressure of $\sim$20 atmospheres does not ensue from this diagram. Kapitza(3) also observed a maximum of the liquefaction coefficient at a pressure of $\sim$17 atmospheres at an initial temperature $\sim 10^0$ K.

As has been shown by Kapitza, this discrepancy between the results of helium liquefaction and the $T—S$ diagram cannot be explained by the drift of liquid drops in the heat-exchanger and, according to Kapitza, is due to the inexactitude of the diagram.

The discrepancy of practical results of helium liquefaction with the Keesom and Hanthoff diagram may be a merely apparent one and result from an inaccurate calculation of the liquefaction coefficient.

The liquefaction coefficient is calculated, as known, according to the equation:

$$\eta = \frac{i_2 - i_1}{i_2 - i_l}, \qquad (1)$$

where i_1 is the enthalpy of the compressed gas admitted to the heat-exchanger, i_2, the enthalpy of the expanded gas let out from the heat-exchanger, and i_l, the enthalpy of the liquid.

The calculation is usually carried out for a given temperature of the expanded gas let out from the heat-exchanger; for an ideal heat-exchange it is assumed to be equal to the temperature of the compressed gas at its admission to the heat-exchanger.

Such an «ideal» heat-exchange is far from being always possible, its possibility being dependent on the course of the isenthalpies in the region of the diagram adjacent to the boundary curve of the liquid.

Let us consider the schematical $T—S$ diagram, represented in Fig. 1. We shall assume that the liquefaction coefficient, calculated according to equation (1), will correspond to the point O on the liquefaction line $a—b$; the possibility of heat-exchange obviously depends upon the point where

the isenthalpy passing through the point O crosses the isobar, corresponding to the pressure of the compressed gas. If this crossing takes place, as shown in Fig. 1, at a temperature below that of the boiling point of the liquid, the heat-exchange is evidently impossible.

The most interesting part of the $T-S$ diagram for helium is unfortunately unknown, but the data presented in the diagram of Keesom and Hanthoff give some reason to suppose that we have here to do with the case considered above.

We see thus that the liquefaction coefficient is not determined unambiguously by the drop of the enthalpy on the hot end of the heat-exchanger, but that it depends also upon the course of the isenthalpies in the region of the diagram, adjacent to the boundary curve of the liquid. If the course of the isenthalpies in this region of the diagram is such as shown in Fig. 1, then in order to make heat-exchange possible and to avail oneself of the total difference of the enthalpies at the temperature corresponding to the admission of compressed gas to the heat-exchanger, resort to special devices is necessary. In substance it would be sufficient to introduce in the scheme of the liquifier an additional throttle-valve (Fig. 2) with the help of which at some intermediate temperature T' a transition occurs from the isobar corresponding to the pressure of the compressed gas p to some isobar p' such that the isenthalpy, corresponding to the liquefaction coefficient calculated according to equation (1), crosses it sufficiently high.

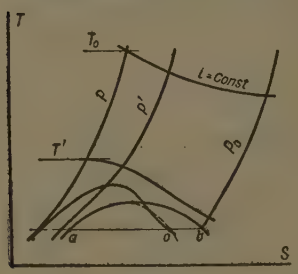

Fig. 1.

Fig. 2.

Fig. 3.

However, as it is difficult to determine beforehand the exact part of the heat-exchanger where the additional throttle-valve should be placed, it is practically more expedient to use the scheme presented in Fig. 3. There a certain portion of the gas is at an arbitrarily fixed temperature T' throttled till one atmosphere, while the other portion, till the pressure P' chosen as mentioned above. The gas expanded in the valve y'' serves for the additional cooling of the gas expanded in the valve y'.

Changing the quantities of the gas passing through the valve y' and y'', it is easy to select a regular working regime.

It should be noted in conclusion that such an additional introduction of valves admits of using pressures of the order of $\sim$60 atmospheres and perhaps oven higher for the liquefaction of helium. In this way liquefaction coefficients of practical value may be obtained when hydrogen boiling under atmospheric pressure is made use of for preliminary cooling.

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PHYSICS

INFLUENCE OF THE RAMSAUER EFFECT ON THE POTENTIAL GRADIENT IN A POSITIVE COLUMN

By B. KLARFELD

(Communicated by S. I. Wawilow, Member of the Academy, 30. III. 1938)

The potential gradient in a positive column has been studied by a number of authors(1). For low pressures in argon, O. Groos' (2) measurements are available, which reveal a highly complicated dependence of the gradient upon pressure. In the present work the discharge in He, Ne, Ar, Kr, Hg-vapours and in K-vapours was investigated in order to find out how the gradient varies with pressure in general. Measurements have been extended as far as possible into the low pressure region. The gradient was measured in a tube of 20 mm diameter; the discharge current varied from 25 to 300 mA. The potential gradient was determined using the zero-current probe method. Due care was exercised to ensure well controlled experimental conditions with regard both to gas purity and measuring method. The results obtained for He, Ne, Ar and Kr with a current of 0.3 A (Fig. 1) are qualitatively similar and change continuously in one direction on passing from light to heavier inert gases. In Fig. 2 data for Hg- and K-vapours are given. The values $p_0 = p\,\frac{273}{T}$, i. e., the pressures referred to 0° C, are plotted along the axis of abscissae.

An analysis of the curves in Fig. 1 makes it evident that the gradient minimum at pressures of about one millimeter of mercury is deeper in heavy rare gases. This fact may possibly be attributed to the Ramsauer effect, which is more pronounced in heavy rare gases.

In order to express the potential gradient in terms of the electron mean free path and of quantities directly measurable experimentally, Langevin's mobility equation was solved simultaneously with the electron drift current equation(3). In the expression obtained

$$E = \frac{i}{0.75 N_e e^2 \lambda_e} \sqrt{\frac{8k \cdot n T_e}{\pi}}$$

where N_e is the number of electrons contained in the positive column per cm of its length, and i, the current in the tube, N_e can be expressed in terms of i_e —the random electron current through the tube cross-section perpendicular to the axis. In this case we obtain substituting $B = \frac{i_e}{i}$:

$$E = \frac{2k}{0.75\pi e} \frac{T_e}{B\lambda_e} = 0.73 \cdot 10^{-4} \frac{T_e}{B\lambda_e} \text{ (in V/cm).} \qquad (1)$$

When deducing equation (1), it was taken for granted that: a) the electrons have a Maxwellian velocity distribution; b) temperature and mobility of the electrons are constant across the section of the column; and c) Langevin's equation is valid for the determination of mobility.

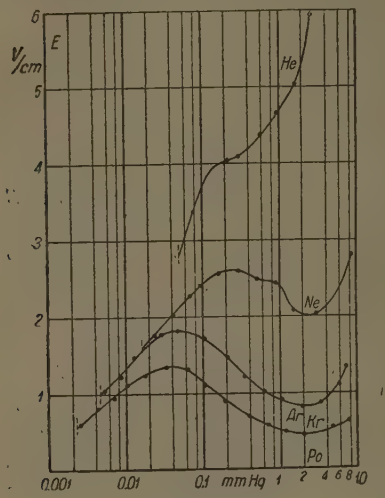

Fig. 1.

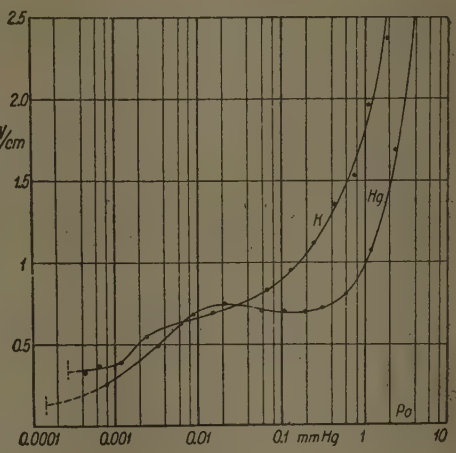

Fig. 2.

To compute E from equation (1), values T_e and B were measured experimentally at different pressures, using the probe method; λ_e was calculated from curves of effective cross-section, viz. electron velocity(4), taking into account an electron velocity distribution corresponding to T_e.

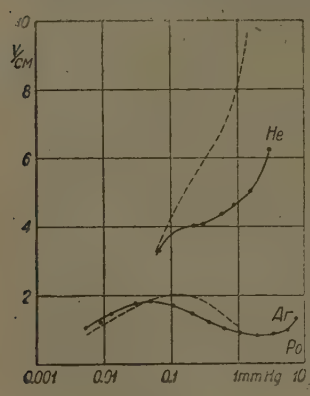

Fig. 3.

T_e and B have been determined experimentally in He and Ar by means of a cylindrical collector along the axis, and by means of a flat one on the wall.

He and Ar were chosen for the investigation, because the Ramsauer effect is absent in He, but pronounced in Ar.

Since λ_e was introduced into equation (1) from Langevin's equation, its reciprocal should be equal to the effective cross-section for impulse transfer rather than to the effective cross-section for collisions(5). Yet calculations derived from data for angular distribution of electrons scattered in collisions with He or Ar atoms (6) show that the difference between the effective cross-section for impulse transfer and that for collisions is insignificant.

In spite of the important rôle played in a positive column by cumulative processes of excitation(7) and ionization, their effect upon λ_e is insignificant too, since the type of collision prevalent under these conditions is the elastic collision(8).

In Fig. 3 experimental curves of the potential gradient in He and Ar are given, as well as those calculated from equation (1). Qualitatively, the curves computed express rather well the type of dependence of gradient upon pressure.

In looking for an explanation of the difference between the gradient curves for argon and helium, we become aware of the fact that for the 20-fold increase of pressure from 0.05 to 1 mm Hg, the change of the value of $\frac{T_e}{B}$ is nearly the same in both helium and argon, while $\frac{1}{\lambda_e}$ increases in helium 26.2 times, and in argon, which exhibits a Ramsauer effect, 5.3 times only.

Thus, in argon the Ramsauer effect is responsible for a deep minimum in the potential gradient curve.

The absence of a gradient minimum in helium and potassium vapours may possibly be connected with a decrease of the effective cross-section of He and K atoms with increasing electron velocity.

In mercury vapours, a slight minimum of the potential gradient is observed. Part of the published data on the dependence of $\frac{1}{\lambda_e}$ upon the electron velocity for mercury vapours[9] points to the presence of a Ramsauer effect, while in many others[10] continuous growth of the effective cross-section with the decrease of electron velocity is reported. It should be noted that earlier measurements of the potential gradient and of the values T_e and B in mercury vapour[11] would be in better agreement with each other, if for mercury there existed a Ramsauer effect for electrons of very low velocities.

The experimental part of this work was carried out under participation of A. F. Asaba.

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PHYSICS

THE PENETRATING COSMIC RAYS PARTICLES AND THE NUCLEAR FORCES

By Ig. TAMM, Corresponding Member of the Academy

The penetrating component of cosmic rays consists most probably of heavy electrons. According to Blackett (1), the cosmic rays particles at sea level are heavy when energetic, but change their mass suddenly when slowed down below a critical energy of about $2—3 \cdot 10^8$ eV, and become ordinary electrons. According to Bowen, Millikan and Neher (2) the penetrating heavies are mainly secondaries produced in the atmosphere by incoming electrons and photons. This conclusion does, however, not necessarily follow from the data of the authors mentioned, these data being consistent also with the assumption of a primary character of the penetrating heavies. At any rate the «transmutations of mass» of the electrons play probably an important rôle in the passage of cosmic rays through the atmosphere.

No adequate mechanism was suggested up to now for these transmutations. Youkawa's hypothesis of a spinless heavy electron seems to lead to difficulties and will not be considered by us. The creation of heavy pairs by photons (3) may be important only for the production of particles of relatively small mass ($m \sim 10\ m_0$) and does not account for the disappearance of the slowed down heavies. The suggestion that the transmutation of mass may occur under absorption or emission of a photon either in vacuum (Jauncey) or under the action of the electric field of a nucleus [Bhabha (4)], implies necessarily the possibility of a spontaneous disintegration of a heavy into an ordinary electron and a photon. It can be shown that any reasonable estimate of the life time of a heavy leads to negligibly small values of the cross-section for the transmutation of mass induced both by photons and by the electric field of the nuclei. Further just as in the case of energy losses by radiation the cross-section for the latter process must be proportional to Z^2, while there are reasons to believe that the cross-section for transmutations is proportional to Z.

Considerations of this kind have led us to the assumption that the heavies are stable in absence of atomic nuclei* and that the transmutation of mass can be induced only by a non-electromagnetic short range interaction of an electron with protons and neutrons.

The simplest way to formulate our hypothesis quantitatively is to introduce in the Hamiltonian of the system: nuclear particles + electrons an interaction operator Ω of the form

$$\Omega = f \sum_{k,l} O \phi_k^* \phi_l \Psi^* \Psi, \qquad (1)$$

* Or, at any rate, that the spontaneous disintegration of heavies during their passage through the atmosphere is negligible.

where f is an universal constant, ψ_k and ψ_l wave functions of electrons of mass km_0 and lm_0 respectively, and Ψ, the wave function of a proton or a neutron. The sum must be taken over all allowed values of k and l, beginning with $k=l=1$. O denotes a Lorentz invariant combination of the wave functions. The deductions from (1) which concern us in this paper will not be substantially altered if one inserts under the sign of the sum in (1) any factor depending on the particular values of the masses km_0 and lm_0.

Let us consider an electron of mass m_i and energy E hitting a nucleus and scattered by it with a modified mass m_s (without any radiation). So long as the recoil of the nucleus may be neglected, the conservation of energy requires that

$$E=c\sqrt{c^2m_i^2+p_i^2}=c\sqrt{c^2m_s^2+p_s^2}, \tag{2}$$

p_i and p_s being the momenta of the incident and the scattered electrons. So long as the wave length of the electron or rather the quantity $\frac{h}{(p_i-p_s)}$ is large as compared with the nuclear radius R (I energy range), the scattering will be isotropic and proportional to the square of the number A of neutrons and protons in the nucleus:

$$\sigma=\frac{f^2A^2E^2}{2\pi h^4c^4}\cdot\frac{p_s}{p_i}. \tag{3}$$

But if $\frac{h}{(p_i-p_s)}$ becomes smaller than R (II energy range), the partial waves scattered by different nuclear particles get out of phase, the forward scattering becomes preponderent and the cross-section σ tends with increasing energy to a constant value independent of E, m_i and m_s and proportional to $A^{4/3}$:

$$\sigma=\frac{9f^2A^{\frac{4}{3}}}{16\pi h^2c^2r_0^2}, \tag{4}$$

where $r_0=RA^{-\frac{1}{3}}\sim 1.4\cdot 10^{-13}$ cm. Finally, when the energy E of the impinging electron exceeds, say, $2\cdot 10^8$ eV (III energy range), the nuclear particles may be considered as free, since the recoil energy imparted to them by the scattered electron exceeds their binding energy (if the angle of scattering is not very small). The cross-section σ will again begin to increase in proportion to E^2. Hence, if only σ has at, say, $E\sim 10^{10}$ eV any reasonably moderate value, its value at $E\sim 5\cdot 10^8$ eV must be quite negligible.

However, in the domain of very high energies the validity of the above calculations becomes questionable. It is well known that an interaction of the type (1) leads to divergent results even in such approximations which can be satisfactorily treated by the present quantum electrodynamics. Thus, if a neutron or a proton can scatter an electron with a changed mass, it is also able to create a pair, the electron and the positron of the pair having eventually different masses (unelastic scattering of an electron, which has had initially a negative energy). It follows, that between neutrons and protons must exist certain forces, corresponding to the exchange of pairs between these particles. But if one tries to evaluate these forces with the help of (1), one gets a divergent result, just as in corresponding calculations in the theory of β-decay*.

* Although the integrations may be arranged in such a way as to give a finite result at finite separation r of the interacting particles, still if one performs the integrations in a somewhat different succession the integrals do diverge even if r remains finite. The reason at the bottom of this is that the interaction corresponds to the exchange of pairs of particles, and not of separate particles (such as photons in the case of the electromagnetic interaction).

For lack of any better theory all one can do is to introduce in the interaction operator (1) some more or less arbitrary «guillotine factor». The simplest and most usual form of this factor is $e^{-\tau(p_k+p_l)}$, where τ is a constant and p_k and p_l denote the momenta of the electrons of mass km_0 and lm_0 measured in that reference system, in which the nucleus is at rest. This factor makes the energy $V(r)$ of the interaction of nuclear particles, which at large distances varies as r^{-5}, to tend to the value

$$V(0) = -\frac{2f^2N^2}{5\pi^4 c\hbar^6\tau^5}[a^2 + b^2(s_1 s_2)], \qquad (5)$$

as r tends to zero. a^2 and b^2 are constants of the order of magnitude 1, depending on the choice of the operator O in (1); s_1 and s_2 are the spin vectors of the interacting particles, and N denotes the number of allowed values of the mass km_0 of an electron. The sign and the spin dependence of V are correct.

But the guillotine factor* effects also the cross-section for the scattering of electrons with transmutation of mass. If $\tau \sim \frac{1}{400\, m_0 c}$, corresponding to a wave length $\lambda \sim 2\pi \cdot 10^{-13}$ cm, the cross-section σ in the I and II energy ranges defined above will be unaffected and will attain at $2 \cdot 10^8$ eV the stationary value (4), but with a further increase of energy σ will begin to decrease exponentially. Hence the probability of transmutation of energetic heavies will be very small, but as soon as the heavies are slowed down to about $3 \cdot 10^8$ eV the value of σ will rapidly increase in accordance with measurements of Blackett. The cross-section for the reverse process (transmutation of an ordinary electron m_0 into a heavier one m_h) is of the same order of magnitude if $E \gg c^2 m_h$, but at lower energies differs from $\sigma(m_h \rightarrow m_0)$ by a factor

$$\frac{p_h^2}{p_0^2} = \frac{E^2 - c^4 m_h^2}{E^2 - c^4 m_0^2} < 1.$$

If one assumes a definite value for τ and for the stationary cross-section of transmutation (4) in air ($\sigma_{\text{stat}}^{\text{air}}$), one can estimate the energy of interaction (5) of nuclear particles. If one puts $\tau = \frac{1}{400\, m_0 c}$ and $\sigma_{\text{stat}}^{\text{air}} = \alpha \cdot 10^{-25}$ cm^2, one arrives at a reasonable value $V(0) = -3\alpha N^2 \cdot 10^6$ eV.

It can be shown that the probability of the creation of a pair by an excited nucleus by means of the mechanism in question is considerably smaller than the probability of the emision of a photon, at least so long as the excitation energy of the nucleus is smaller than, say, $5 \cdot 10^6$ eV.

The possibility to explain the nuclear forces by the exchange of pairs between neutrons and protons has already been suggested by Teller ([5]) and by Wentzel ([6]), who however considered only ordinary electrons ($m = m_0$) and were unable to establish any connection between the proposed explanation of nuclear forces and other physical phenomena. As pointed out by these authors, the forces due to the exchange of pairs do not depend on the electric charge of the nuclear particles and are consistent with Wick's explanation of the magnetic moment of the neutron. The only serious difficulty is that they apparently do not possess saturation properties.

In conclusion it may be noted that some of the cloud chamber tracks showing an apparent increase of energy of a cosmic particle may possibly be due not to accidental disturbances or to upwardly directed cosmic rays,

* This factor accounts in some very crude manner for the finite size of the interacting particles. Such a factor is superfluous in the case of Coulomb scattering, since for the latter only relatively distant encounters are essential.

but to the transmutation (taking place in the absorbing plate, especially if it is made of a light material such as Al) of a heavier electron into a lighter one, these transmutations being connected according to (2) with an increase of the momentum p.

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PHYSICS

HEAVY ELECTRONS IN COSMIC RAYS

By V. VEKSLER and N. DOBROTIN

(Communicated by S. I. Wawilow, Member of the Academy, 8.V.1938)

The properties of the penetrating component of cosmic radiations induced a number of authors (1) to believe that this component consists of particles possessing a relatively big mass. The study of the nature of these particles and of the mechanism of their production has of late acquired quite particular interest.

For a study of this question it appears to be necessary not only to effect measurements of the number and the absorbability of these particles, but also to know of the ionization they are liable to produce. Accordingly, so-called proportional counters (2) are used to great advantage for this purpose.

The first experiments conducted on mount Elbrouz in the summer of 1937 by Veksler and Isajev permitted to ascertain the presence of strongly absorbed particles in cosmic rays (3). But these experiments did not yet yield sufficient evidence for judging of the nature of these strongly absorbed particles. The present work forms a continuation of Veksler's and Isajev's experiments conducted with the same experimental arrangement at sea level.

I. Description of the Experimental Arrangement

The main elements of the whole arrangement were composed of two groups each of them consisting of five proportional counters connected in parallel and located one beside the other. The cylindrical cathods of the counters were made of metal wire gauze. Each group of counters was located in an earthed metallic box with windows in its bottom and lid, these windows being covered with foil. The boxes were placed one above the other. The counters were at atmospheric pressure. The air equivalent of the foil and the clearance between the groups of counters was $\approx$11 cm. The anodes of the counters of each group were connected with the branches of a Johnson amplifier with preliminary amplification. The whole arrangement together with the high tension battery was placed in a screened room. The sensitivity of the arrangement could be adjusted by varying the tension in the counters or by changing the amplification coefficient at the radio circuit. The sensitivity of the radio circuit in the absence of gas amplification was measured with the help of α-particles. The coefficient of gas amplification in the counters was also measured with the help of α-particles (by means of compensating gas amplification by a several-fold decrease of the radio amplification). Besides this, gas amplification was also measured directly by means of determining

the magnitude of the deviation of the electrometer filament from α-particles. For control purposes, the lower limit of gas amplification was determined by the ratio of gas-amplified current to saturation current induced in the counters by γ-rays of Ra. The results of these measurements gave practically the same magnitude of the coefficient of gas amplification; this coefficient was found to equal 200—500. Radio amplification was such that with maximum sensitivity the numerator connected to the amplification circuit operated when 300—600 pairs of ions were formed in the counters (i. e. with the simultaneous fall in of no less than 3—6 electrons).

It was found while using the above described apparatus that both at sea level and on Mt. Elbrouz the number of coincidences in the counters was liable to decrease markedly if a thin iron plate was placed between them. In order to make sure that the coincidences observed were indeed connected with cosmic rays, the control experiments effected on Mt. Elbrouz were repeated again. It was found that even very strong impulses in one circuit do not cause operation of the numerator in the coincidence scheme. Besides this, a decrease of tension by 200 V in one group of counters was found to be sufficient for a complete elimination of all coincidences. This showed that the operation of the numerator was caused by the appearance of discharges in the counters and was not due to any other causes whatsoever.

In order to prove that coincidences eliminated by the insertion of plates cannot be due to groups of slow electrons, or else to radioactive contaminations, special experiments were set up with the irradiance of counters with Ra. The sensitivity of the apparatus in these experiments was somewhat greater than normal operating sensitivity. The intensity of γ-rays from Ra exceeded several times the intensity of the same rays from an usual radioactive contamination of the laboratory. Experiments have shown that though the number of impulses in each group of counters increased in course of radium irradiance one and a half time while the general number of coincidences grew 1.2 times, yet the number of coincidences eliminated by plates remained practically unchanged.

If we were to take it for granted that the increase of the full number of coincidences is determined by mere chance coincidence of discharges in both groups of counters, then we obtain for resolving power τ the magnitude $\sim 3.3 \cdot 10^{-4}$ seconds. Direct measurements of τ effected by means of the determination of chance coincidences (both groups of counters are extended in the horizontal plane) gave for τ the magnitude $\sim 4 \cdot 10^{-4}$ seconds, i. e. we obtain in this case practically the same magnitude as with experiments with Ra.

Thus, from the control experiments carried out it follows that the coincidences eliminated by plates are, as a matter of fact, brought about by cosmic radiation.

II. Results of Measurements

The dependence of the number of coincidences upon plate thickness was investigated with the described apparatus, the plate being placed between the upper and the lower group of counters. The measurements were effected with three different sensitivities. The results of the measurements are given in Fig. 1. Here, along the axis of the ordinates are plotted the number of coincidences in one minute referred to an area of 1 cm^2 occupied by a single group of counters, while along the axis of abscissae is to be found the thickness of the plates; in obtaining Curve *I* the arrangement registered coincidences in every group of counters where >300—600 pairs of ions were arising; Curve *II*—by 400—800 pairs of ions; Curve *III*—by 700—1400 pairs of ions.

As will be seen from Fig. 1, a large number of particles (or groups of particles) has a range of the order of some tenths of g/cm². From confronting Curves *I* and *III* it follows that a large number of such particles sets up in the groups of counters more than 400 but less than 1000 pairs of ions. The data given in Fig. 1 represent the mean figures from a number of observations*. The points corresponding to one and the same thickness of the plate were obtained for all three curves by means of varying radio amplification and were fixed consecutively for all three curves. The curves obtained on different days and in various ways could, all of them, be well reproduced. On the whole 655 coincidences were observed for Curve *I*, 349 coincidences for Curve *II*, and 167 coincidences for Curve *III*.

Further on an investigation was made of the dependence of the number of particles absorbed by the plate (0.53 g/cm²) upon the thickness of lead located above the counters (Curve *I*, Fig. 2). Curve *II* gives the same dependence for coincidences which cannot be eliminated by plates. As was to be expected, this curve is virtually nothing

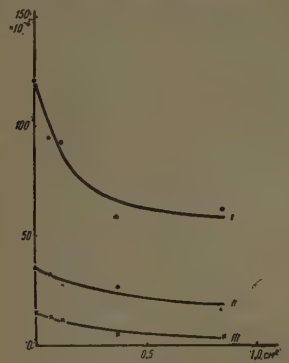

Fig. 1.

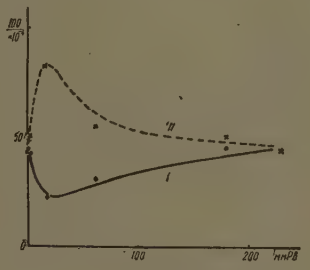

Fig. 2.

but the well-known Rossi-curve for ordinary showers. In these experiments the total number of coincidences without plate was 2222; with plate—1464. The results obtained on different days were fully reproducible.

From Fig. 1 and 2 it follows that the data obtained agree qualitatively with the preliminary observations effected on Mt. Elbrouz.

III. Discussion of Results

A comparison of Curves *I* and *II* in Fig. 2 shows conclusively that such coincidences as can be eliminated by plates cannot be caused by slow electrons connected with usual showers. These coincidences that can be eliminated by plates can neither be explained by the chance appearance of a few δ-particles formed by particles with great energy. It is possible to show that the mean number of secondary electrons with energies of several hundreds of kilovolts accompanying a single primary particle will be of the order of 0.1. Therefore, the probability of a chance appearance of a group consisting of several such particles will tend to drop with extreme rapidity with the increase of the number of particles in a group. Thus, in this case the number of coincidences that can be eliminated by plates should depend on the sensitivity of the apparatus to an extent which can even not be compared to that observed in reality. Besides this, the number of δ-particles and, consequently,

* The curves are made with due allowance for chance coincidences, coming to 15—25% of the total number of all coincidences.

of coincidences that can be eliminated by plates should increase considerably, if some dense substance be placed immediately above the counters. Experiments with lead and qualitative observations conducted with textolite (0. 7 g/cm^2) have shown that this conclusion too is markedly conflicting with experimental data.

From this it follows that coincidences that can be eliminated by plates can by no means be attributed to the action of slow electrons arising in the course of certain processes of interaction between cosmic radiation and matter. It is also difficult to suppose that in this case we are dealing with showers of a quite special kind*.

Therefore, coincidences that can be eliminated by the use of plates should be attributed to the action of strongly ionizing heavy particles. The mass of these particles may be evaluated by comparing their range and the ionization that they produce. It is well known that ionization is determined by the velocity of particles and does not depend on mass. This circumstance provides the possibility of estimating the velocity of particles from the number of pairs of ions produced in the counters. Then, from the range and velocity of the particles it is possible to obtain an approximate value of the mass. The estimations made in this way give, for the mass of the observed particles, a value exceeding the mass of the electron 50—300 times. This shows that these particles are not protons but «heavy electrons», observed in cosmic rays by a number of recent researches.

The circumstance that the number of «heavy electrons» does not decrease (and even tends to grow) with great thickness of lead placed above the counters, shows that these particles are in some way connected with the penetrating component of cosmic radiation. However, we are not dealing in this case with the ends of the range of «heavy electrons» which, according to latest evidence, constitute the penetrating component itself. This follows from the fact that in this case the absorption coefficient of the penetrating component which may be easily determined from the data of Fig. 1, would have been about 10 times greater than the figure yielded by direct measurements. Accordingly, we have to draw the conclusion that the «heavy electrons» investigated are secondary (or ternary, etc.) particles produced by the penetrating component of cosmic radiation as a result of some kind of interaction with matter.

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* This question may be solved by a series of special experiments being prepared at present in which use will be made of the Wilson chamber controlled by proportional counters.

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PHYSICAL CHEMISTRY

KINETICS OF WETTING AND SELECTIVE LINEAR CORROSION OF METALS IN POLYPHASE SYSTEMS: METAL-LIQUID-LIQUID AND METAL-LIQUID-GAS. V

By D. I. MIRLIS*

(Communicated by V. A. Kistjakowskij, Member of the Academy, 31. III. 1938)

This paper, like the previous one, has for object to investigate the influence of the length of the hydrocarbon chain of saturated fatty acids (HCOOH, $CH_3.COOH$, $CH_3.CH_2.COOH$, $CH_3.CH_2.CH_2.COOH$), distributed in two liquids of different polarities (water—benzene), on the kinetics of wetting and on the effect of selective linear corrosion of aluminium along the phase boundary line and within each phase separately.

Experimental

The method I previously used in investigating the influence of the length of the hydrocarbon chain of fatty acids on the kinetics of wetting and on the selective linear corrosion of aluminium along the three-phase boundary line has been described in I and IV([1]). The oxide film on aluminium is hydrophilic in contrast with iron, as it has been pointed out by us in communication III([1]) and will be experimentally shown below. Trimming of the surface of aluminium with a dead smooth file, after our method, or trimming with a subsequent activation with corrosive sublimate, alters the nature of aluminium surface making the latter comparatively hydrophobic. This circumstance favours the study of the comparative effect of selective linear corrosion of Fe and Al under the influence of single fatty acids. In addition to the above mentioned, experiments were made with aluminium having a smooth surface (rolled aluminium). Using these methods the following equilibrium systems were studied:

1. Al—norm. aq. sol. of HCOOH—benzene	Rough surface
2. Al— » » » » $CH_3 \cdot COOH$—benzene	Rough surface
3. Al— » » » » $CH_3 \cdot CH_2 \cdot COOH$—benzene	Rough surface
4. Al— » » » » $CH_3 \cdot CH_2 \cdot CH_2 \cdot COOH$—benzene	Rough surface
5. Al— » » » » $CH_3 \cdot CH_2 \cdot CH_2 \cdot COOH$—benzene	Rough surface activated with 0.01% $HgCl_2$ solution
6. Al— » » » » $CH_3 \cdot CH_2 \cdot CH_2 \cdot COOH$—benzene	Smooth surface
7. Al— » » » » $CH_3 \cdot CH_2 \cdot CH_2 \cdot COOH$—air	Smooth surface
8. Al—water—benzene	Rough surface activated with 0.01% $HgCl_2$ solution

* E. P. Litvinova, Senior Laboratory Assistant, participated in the experiments.

Table 1

Time in minutes	Rough surface of aluminium								Rough surface of Al activated with $HgCl_2$		Smooth surface of aluminium				Rough surface of Al activated with $HgCl_2$
	Al — norm. aq. sol. of HCOOH—benzene		Al—norm. aq. sol. of CH_3·COOH—benzene		Al—norm. aq. sol. of CH_3·CH_2·COOH—benzene		Al—norm. aq. sol. of CH_3·CH_2·CH_2·COOH—benzene		Al—norm. aq. sol. of CH_3·CH_2·CH_2·COOH—benzene		Al—norm. aq. sol. of CH_3·CH_2·CH_2·COOH—benzene		Al—norm. aq. sol. of CH_3·CH_2·CH_2·COOH—air		Al—water—benzene
τ	Θ_{12}	B_{12}	Θ_{12}	B_{12}	Θ_{12}	B_{12}	Θ_{12}	B_{12}	Θ_{12}	B_{12}	Θ_{12}	B_{12}	Θ_{12}	B_{12}	Θ_{12}/B_{12}
0	90	0.000	113	−0.391	135	−0.707	130	−0.643	159	−0.934	120	−0.500	28	+0.883	The contact angle cannot be measured because the jet of violently evolving hydrogen thrusts the drop of water away
1	72	+0.309	94	−0.069	135	−0.707	130	−0,643	159	−0.934	115	−0.423	26	+0.889	
3	65	+0.423	84	+0.105	131	−0.656	130	−0.643	159	−0.934	113	−0.391	26	+0.899	
5	60	+0.500	80	+0.174	121	−0.515	130	−0.643	159	−0.934	111	−0.358	26	+0.899	
9	55	+0.573	75	+0.259	121	−0.454	130	−0.643	159	−0.934	107	−0.292	25	+0.906	
17	52	+0.616	66	+0.407	117	−0.391	130	−0.643	159	−0.934	105	−0.259	24	+0.914	
34	50	+0.656	65	+0.423	113	−0.309	130	−0.643	159	−0.934	104	−0.242	23	+0.920	
64	48	+0.669	62	+0.469	108	−9.309	130	−0.643	159	−0.934	101	−0.191	19	+0.945	
124	47	+0.688	60	+0.500	108	−0.309	130	−0.643	159	−0.934	98	−0.139	10	+0.985	
184	46	+0.695	60	+0.500	108	−0.309	130	−0.643	159	−0.934	98	−0.139	—	—	
240	33	+0.839	56	+0.559	108	−0.309	130	−0.643	159	−0.934	97	−0.122	—	—	
360	22	+0.927	—	—	103	—	—	—	159	−0.934	97	−0.122	—	—	
480	10	+0.985	51	+0.629	—	—	128	−0.615	159	−0.934	90	0.000	—	—	
24 hours	—	—	48	+0.669	76	+0.241	98	−0.139	159	−0.934	81	+0.156	—	—	
48 »	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
72 »	—	—	—	—	—	—	166	−0.970	159	−0.934	—	—	—	—	

The results obtained in measuring the kinetics of wetting $B_{12}=f(\tau)$ are given in Table 1.

On comparing the data obtained with aluminium in the present investigation (Fig. 1) with the results found with iron and reported by the authors in communication IV, it will be seen that all the curves drawn for the kinetics of the corrosion process on aluminium under the influence of the fatty acids lie on the diagram of wetting somewhat below the ones for iron. The effect of selective linear corrosion on aluminium under the conditions of our experiments was feeble in contrast with our data obtained with a normal solution of HCl (1). The processes that take place on aluminium, having on its surface a drop of an aqueous solution of butyric acid and immersed in benzene saturated with the same acid (equilibrium system), attract considerable attention. Thus, in the first 500 minutes the contact angle (θ_{12}) of wetting remains nearly constant ($\theta_{12}=128$—$130°$). Then the aluminium surface becomes more and more hydrophilic and after 24 hours from the beginning of the experiment the contact angle turns into $\theta_{12}=98°$. After that the contact angle begins increasing, i. e. the drawing together of the drop of the «aggressor» with time is taking place and after 72 hours from the beginning of the experiment the contact angle grows up to 166°. This indicates a strong hydrophobization of the aluminium surface. This phenomenon, in all probability, is due to a reaction of the fatty acid with the product of corrosion(1). A similar phenomenon was observed by the author on duralumin (initial — 125°, after 24 hours—80°, after 48 hours—130°).

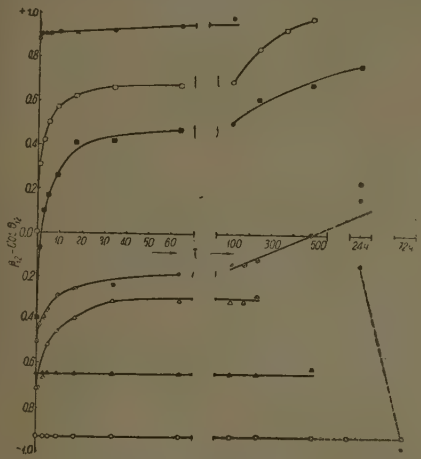

Fig. 1. Length of carbon chain of fatty acids as affecting the kinetics of wetting and the local selective corrosion of aluminium at the phase boundary line.

Rough surface of aluminium

□ Al — norm. aq. sol. of HCOOH — benzene
■ Al — » » » » CH_3COOH — benzene
△ Al — » » » » CH_3CH_2COOH — benzene
▲ Al — » » » » $CH_3CH_2CH_2COOH$ — benzene
○ Al (activated with $HgCl_2$) — norm. aq. sol. of $CH_3CH_2CH_2COOH$ — benzene

Smooth surface of aluminium

⊙ Al — norm. aq. sol. of $CH_3CH_2CH_2COOH$ — benzene
⊕ Al — » » » » $CH_3CH_2CH_2COOH$ — air

After the activation of aluminium surface with a 0.01% $HgCl_2$ solution (2) during 15 sec. and a subsequent depositing of an adsorption layer of 0.2% oleic acid from benzene, the contact angle of wetting becomes near 180°, i. e. the aluminium surface acquires a limit of hydrophobity (a water drop rolls off such a surface).

On depositing an adsorption layer of butyric acid on the activated aluminium surface, the latter, as it may be seen from Table 1, becomes more hydrophobic ($\theta_{12}=159°$) in comparison with the rough inactivated surface. After a more prolonged activation of the aluminium surface with corrosive sublimate or on increasing the concentration of the latter above the one indicated, no adsorption layer will be formed on the surface of the metal.

The layer of the surface active substance having a short hydrocarbon chain—the lower member of the homologous series, as, for instance, propionic acid—is permeable to the «aggressor» (3). The latter, percolating under the

Table 2

Al—norm. aq. sol. of R.COOH—benzene	Mean loss of Al in mg per 193 days (Δm)		Mean loss of Al in mg per 1 cm² of the surface ($\frac{\Delta m}{S}$)		Nature of the corrosion of Al within the individual phases and along the phase boundary line		
Equilibrium systems (rough surfaces)	Aq. phase	Benzene phase	Aq. phase	Benzene phase	Aq. phase	Benzene phase	Phase boundary
Al—norm. aq. sol. of HCOOH—benzene	66.2	25.1	10.5	3.4	Uniform corrosion with evolution of hydrogen	Local affections over the whole surface	Effect of the linear corrosion absent
Al—norm. aq. sol. of CH_3·COOH—benzene	17.4	2.2	2.8	—	Corrosion chiefly near the boundary line	The effect of corrosion practically absent	Linear corrosion takes place
Al—norm. aq. sol. of $CH_3 \cdot CH_2$·COOH—benzene	37.3	3.6	5.9	—	Uniform corrosion over the whole surface	Individual local affections near the boundary	Local affection of the metal. Linear effect absent
Al—norm. aq. sol. of $CH_3 \cdot CH_2 \cdot CH_2$·COOH—benzene	89.4	20.2	14.2	3.2	Intense corrosion near the boundary	Intense corrosion near the boundary	Intense local destruction of the surface (Fig. 2)

Table 3

Homogeneous Systems

Al—benzene solution of R·COOH	Mean loss in weight of Al in mg per 186 days	Nature of surface after the corrosion
Al—benzene solution of HCOOH	8.4	3—4 local affections of the surface
Al—benzene solution of CH_3·COOH	1.8	Destruction practically absent
Al—benzene solution of $CH_3 \cdot CH_2$·COOH	2.6	
Al—benzene solution of $CH_3 \cdot CH_2 \cdot CH_2$·COOH	1.9	

film, lifts it in result of an intensive process of corrosion and throws the drop of the «aggressor» off. Under an influence of distilled water the process of corrosion on an active aluminium surface goes on in violent manner, so that the contact angles cannot be measured in this case, as in the previous one. On the smooth burnished surface of aluminium (rolled aluminium), which is more hydrophilic, there is, in contrast with a trimmed rough surface of aluminium, no chemical fixation of an adsorbed layer of fatty acid, as it will be shown below. The drop does therefore spread without any visible injury to the oxide film (Table 1), in accordance with the data of P. A. Rehbinder and his collaborators([4]). That the nature of the oxide film on aluminium differs qualitatively from the oxide film on iron, has been demonstrated by the author likewise by means of an electrochemical method. Certain interest in this aspect have the studies of N. A. Isgarishev, H. Sutton and E. Schmidt concerning the chemical composition and structure of the film on aluminium surface, and likewise the papers by G. Akimov ([5]).

Fig. 2. Equilibrium system: Al—water solution of butyric acid—benzene. The corrosion process is intense at the phase boundary line. A nearly throughout destruction.

Along with the data above, experiments were conducted to investigate the process of corrosion under the conditions of a polyphase contact in presence of fatty acids, according to the macro-method described by the author in communication IV. For this research an aluminium wire (Kahlbaum), 3 mm in diameter, was used. From this wire the collapsible metal cylinders had been prepared to enable the registering of the loss of aluminium in every phase separately. The qualitative observation of the process of corrosion during 192 days has shown all the course of the process of corrosion, making apparent a long induction period, precedent to the corrosion itself (20—30 days) (Table 2).

The process of corrosion was going on principally along the phase boundary line, the corrosion along the boundary growing more intensive as the length of the hydrocarbon chain increased (Fig. 2). On iron the reverse phenomenon takes place under the same conditions. In the benzene phase aluminium compared with iron is but slightly exposed to corrosion. On comparing the experimental data obtained in this research and likewise the data on the subject from the author's previous investigations, a conclusion can be reached: the effect of the selective linear corrosion along the three-phase boundary line depends largely, together with other factors (influence of pH of the solution([8]), the condition of the surface, etc.), on the formation of the fourth phase along this boundary in the course of the corrosion process. This fourth phase is a solid, insoluble in both liquid phases, product of corrosion. It means that the effect of corrosion depends largely on the solubility or insolubility of the solid corrosion product in the liquid phases([6,7]), as well as in the absence of condition for the removal of the evolved hydrogen gas from the metal. The data obtained are of a particular interest for the conciliation of the theory of differential aeration with the effect of the selective linear corrosion([6]). The effect of the selective linear corrosion studied by the author is in full agreement with the classification of V. A. Kistjakovskij ([7]).

The benzene solution of fatty acids was obtained by saturating the benzene over a fivefold volume of a normal aqueous solution of these acids.

Aluminium suffers the greatest local destruction near the phase boundary line (Fig. 2).

During 6 months aluminium has shown no visible changes in a homogeneous hydrocarbon solution of a fatty acid.

From this data compared with the data for other metals we can draw a conclusion that the removal of the factor of polyphasity, i. e. the removal of moisture([6], [1,7]), consisting in the form of an independent phase, from the liquid hydrocarbon fuel greatly reduces the losses of metal through corrosion and at the same time removes the most dangerous kind of local corrosion.

Summary

1. It has been shown that the stronger resistance of aluminium, compared with iron, to corrosion in the benzene and aqueous phases of a polyphase system containing fatty acids is connected with the nature of the oxide film on aluminium. In particular it is due to its considerable hydrophility, which makes it qualitatively different from the oxide film on iron.

2. It has been experimentally shown that on an aluminium surface freed from the oxide film (moderately activated by 0.01% solution of corrosive sublimate) a chemically fixed adsorbed layer of the surface active acids (butyric, oleic) is formed. No such phenomenon takes place in the presence of an oxide film.

3. In previous communications, as in the present one, it has been shown, that the moisture, insoluble in liquid hydrocarbon fuel and forming an independent phase, appears (particularly in the presence of an electrolyte) as a principal aggressor in the systems studied by the author. The moisture stimulates chiefly the effect of the selective linear corrosion of metals on the three-phase boundary line.

4. It has been shown that the removal of moisture, forming an independent phase in the system metal-liquid-liquid and creating the polyphasity of the system, greatly reduces the losses due to corrosion removing at the same time the most dangerous kind of local corrosion.

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CHEMISTRY

STRUCTURE OF ASSOCIATED MOLECULES OF ORGANIC ACIDS ON THE BASIS OF RAMAN SPECTRA

By V. ČELINZEV, Corresponding Member of the Academy

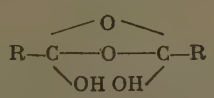
It is well known that organic acids present associated systems not only at ordinary temperatures, but also in the vapour phase and in indifferent solvents, forming complexes in form of dimers, as it was already shown by Auwers(1).

As the dipolmoments of these associated molecules in solutions of non-polar solvents, such as benzene, are nearly zero, according to determinations of Smyth and Rogers(2), the conclusion may be drawn that such associated «doublet forms» must be systems having a symmetrical structure.

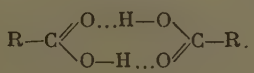
In a pure state these acids have a certain, though not very high, dipolmoment, for instance $H \cdot COOH$—1.5; $CH_3 \cdot COOH$—1.7; $C_3H_7 \cdot COOH$—0.93.

In a water—or ether—solution, as in polar solvents, these acids dissociate into simple monomer molecules, as it was shown by Smyth and Norton(3).

There exist two hypotheses concerning the structure of dimeric associated molecules: according to one hypothesis the following form, which may be called acetal-form, is accepted:



According to the other, special complex forms with a so-called «hydrogen bond» are proposed(4).



But neither of the proposed schemes is acceptable. The first one, the acetal-form, disagrees with the easiness with which dimers can dissociate and deprives the associated molecules of acids of their acid properties (makes them neutral compounds similar with disaccharides). Neither does it agree with their Raman spectra which deny the presence of OH-groups in such dimers. The second form, as a matter of fact, has recourse to the allowance of the bivalency of hydrogen.

The second scheme has been lately the object of particular debates. Zahn(5), whose data agree with the measurements of the electron diffraction made by Hengstenberger and Bru(6), comes to conclusions which disagree with

this scheme. On the contrary, Sannié and Poremski, in a recent paper(7), approve the scheme from their investigations of the Raman effect.

Sannié and Poremski have stated that for acids dissolved in benzene or hexane oscillations 1640—1656 are characteristic. For acids in a free state the oscillations are 1670, while in water, alcohol or ether solutions they are 1712, 1706 and 1750 respectively.

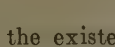
Data in benzene and hexane solutions, in which the acids are completely associated into dimers, have been reported by Dadieu and Kohlrausch(8). Elsall(9) has published data for pure acids; Dadieu and Kohlrausch—for acids in polar solvents, Sannié and Poremski—for acids with 25 per cent of water, corresponding to the complex $CH_3COOH \cdot H_2O$, have given the figure 1712.

These authors compare the Raman spectra for acids with the spectra for ketones and aldehydes having oscillations 1714—1715, and for ethers 1736, and come to the conclusion that the figure 1750 for acids corresponds to the group CO, and the figures 1706—1712 correspond to the complexes $R \cdot COOH \cdot H_2O$. The figure 1640 can be found only in those cases when the molecules of acids are associated into dimers. This emphasizes the fact that in dimers there appear special structures, i. e. that the group CO, as such, ceases to exist and oscillations corresponding to the OH-group and responding to the figure 1412 disappear at the same time too.

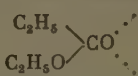
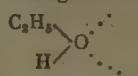
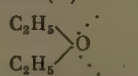
All these facts lead the author to the above-mentioned scheme II having a hydrogen bond. In this scheme there is neither a simple CO-group, nor a free OH-group and therefore oscillations 1750 and 1412, peculiar to these groups, will be absent, but there will be new oscillations 1640, corresponding to the new dimeric condition of the molecules.

We think that the course of reasoning of the above authors, which led them to the conclusion, based on the investigation of the Raman spectra, about a peculiar condition of carboxyl groups in dimers, is correct, but that the scheme of associated molecules proposed on the base of these reasonings is incorrect.

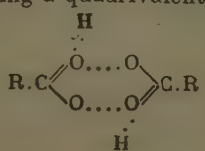
As the base for the construction of formulae for the dimers of acids the existence of higher valencies of carboxyl groups should be accepted, with the possibility of the locking with these groups of mobile hydrogen atoms belonging to their hydroxyl groups, according to the scheme:



We consider that the existence of strong higher valencies of carboxyl groups has been proved experimentally, and on the basis of our thermochemical investigations we may say that these valencies are displayed with much more strength than the higher valencies of hydroxyl or oxide-oxygen atoms. This may be seen from the thermal effect of the formation of oxonium complexes with individual organo-magnesium alcoholates(10).

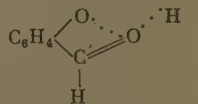
 12 Cal;  11 Cal;  5 Cal.

On the base of these statements, we think that it would be most natural to express the structure of dimeric complex molecules of acids by means of the following scheme having a quadrivalent oxygen:

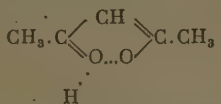


Lately M. and R. Freymann[11] have paid attention to the fact that in the Raman spectra of salicylic aldehyde, of enol isomers of diketones in alcohol solution and of phenols in ether solution, in dioxane, in acetone, the oscillations characteristic for OH-groups are totally absent. This may be drawn from direct experiments and from determinations of many other authors.

For all these cases the following schemes may be proposed which are similar to the previous one, but only intramolecular:



Salicylic aldehyde



The enol form of diketones.

The existence of acid salts of monobasic acids can be explained by means of similar schemes.

The formulae proposed by us give an exhaustive explanation why in certain cases oscillations corresponding to hydroxyl groups and simple carbonyl groups are absent in acids [at any rate, these oscillations are very much weakened, as it was shown by many investigators[12]]. In these cases new oscillations appear in the region of the spectrum 1640, characteristic for the new groupings of atoms, corresponding to associated molecules.

In agreement with our former paper[13] it should be accepted that in layers on water surface the above-mentioned systems partly dissociate and give associations with molecules of water, forming a layer where carboxyl groups are directed towards molecules of H_2O. This opinion may be confirmed by Raman spectra: in water solutions of acids there appear again oscillations having a wavelength 1712—1750, corresponding probably to complexes of acids and water and to dislocked or dissociated molecules of acids.

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CHEMIE

ÜBER DAS GESETZ DER PERIODIZITÄT

Von P. PETRENKO-KRITSCHENKO, korrespondierendem Mitglied der Akademie, und von J. EKSTER

ÜBER DIE UNTERBRECHUNG DER KONTINUITÄT IN DEN WERTEN DES MOLEKULAREN DREHUNGSVERMÖGENS HOMOLOGER REIHEN OPTISCH AKTIVER SUBSTANZEN

Der numerische Wert des Drehungsvermögens ist nicht allein von der Temperatur, der Konzentration und der Natur des Lösungsmittels, sondern auch von der chemischen Struktur des Moleküls der betreffenden Substanz in bestimmter Weise abhängig.

Die Frage nach der Ermittelung bestimmter Gesetzmässigkeiten in der Abhängigkeit des Drehungswertes von der chemischen Struktur beschäftigt schon seit langem viele Forscher.

Durch die Arbeiten von Landolt, Walden und Tschugajew wird eine ganze Reihe diesbezüglicher Hypothesen und Gesetzmässigkeiten aufgestellt. Es sind jedoch bisher nur wenige befriedigende Resultate zu verzeichnen. Diese Hypothesen und Regeln weisen eine Reihe von Ausnahmen auf, sogar in den Beispielen, deren sich die Autoren zur Bekräftigung der von ihnen angegebenen Gesetzmässigkeiten bedienen.

In vorliegendem Aufsatze wollen wir versuchen, die Richtigkeit der vorhandenen Hypothesen zu untersuchen, und zwar in bezug auf:

1) die Veränderungen des Wertes $(M)_d$ in den homologen Reihen optisch aktiver Substanzen;

2) die Veränderungen des Wertes $(M)_d$ in Abhängigkeit vom Sättigungsgrad der Gruppe.

1. Veränderung des Wertes des molekularen Drehungsvermögens $(M)_d$ in homologen Reihen

Auf Grund der Beispiele, die von Guy, Walden, Frankland u. a. angeführt werden, zieht Landolt[2] den Schluss, dass der Wert $(M)_d$ in den homologen Reihen von Estern mit der Steigerung des Molekulargewichts des Alkoholradikals entweder zu- oder abnimmt.

Bei Betrachtung der Veränderungen des $(M)_d$-Wertes in den homologen Reihen der Ester von Menthol und Borneol, kommt Tschugajew[3] zu folgendem allgemeinem Schluss:

«Von einem bestimmten Serienglied an hört die weitere Zunahme des Molekulargewichts des Substituenten auf, einen merklichen Einfluss auf den $(M)_d$-Wert auszuüben, und es bleibt letzterer annähernd konstant».

Wollen wir nun erst diese Schlüsse an von den Autoren selbst und dann von anderen Forschern angeführten Beispielen überprüfen.

			$(L)_d$	$(M)_d$
1) ([3])	Methyläther	der Glycerinsäure	— 4.8	— 5.76
	Äthyläther	» »	— 9.18	— 12.3
	Äthylpropyläther	» »	— 12.94	— 19.15
	Äthylbutyläther	» »	— 18.19	— 23.37
	Heptyläther	» »	— 11.3	— 23.05
	Oktyläther	» »	— 10.22	— 22.28
2) ([4])	Methyläther	der Dibenzoylglycerinsäure	26.89	88.2
	Äthyläther	» »	26.58	90.9
	Äthylpropyläther	» »	21.0	74.76
3) ([5])	Methyläther	der Äpfelsäure	— 6.85	— 11.1
	Äthyläther	» »	— 10.18	— 19.35
	Äthylpropyläther	» »	— 11.62	— 25.32
	Amyläther	» »	— 9.92	— 27.19
	Kapryläther	» »	— 6.92	— 24.77
4) ([6])	Methyläther	der Chlorbernsteinsäure	41.42	74.8
	Äthyläther	» »	27.5	57.8
	Äthylpropyläther	» »	25.63	60.6
	Äthylbutyläther	» »	21.57	57.1
	Amyläther	» »	21.56	63.1

Wie aus den obigen Beispielen zu ersehen ist, haben wir keine kontinuierliche Ab- oder Zunahme des Wertes $(M)_d$ bei der Zunahme des Molekulargewichts des Alkoholradikals: in allen diesen Beispielen nimmt der Wert des $(M)_d$ mit dem Anwachsen des Molekulargewichts des Radikals des Alkohols tatsächlich zu oder ab, es tritt jedoch ein bestimmtes Maximum oder Minimum auf, nach denen eine weitere Ab- oder Zunahme des Wertes des $(M)_d$ folgt.

Im Beispiele Nr. 4 haben wir zwei Minima (57.3 und 57.1), nach welchen eine weitere Zunahme des Wertes $(M)_d$ kommt.

Tschugajew ([7]) leitet sein Prinzip der Konstanz des molekularen Drehungsvermögens auf Grund der nachstehenden homologen Reihe her:

			$(M)_d$
5)	Ameisenäther	des Menthols	146.3
	Essigäther	» »	157.3
	Propionäther	» »	160.2
	Oläther	» »	156.9
	Valerianäther	» »	157.3
	Kapronäther	» »	157.7
	Heptyläther	» »	157.7
	Kapryläther	» »	155.8
	Napiläther	» »	157.3

Tschugajew nimmt an, dass $(M)_d$ in dieser Reihe, beginnend mit dem Essigäther des Menthols, nahezu keine Veränderung aufweist.

Es will uns scheinen, dass, wenn man $(M)_d$=157.3 für $(M)_d$=157.7 gleich halten kann, so darf man aber durchaus nicht $(M)_d$=160.2 gleich $(M)_d$=155.8 annehmen.

Wir haben in dieser homologen Reihe ein Maximum (160.2), dem eine Abnahme folgt, und ein Minimum $(M)_d$=155.8, dem eine Zunahme des Wertes des molekularen Drehungsvermögens folgt.

Nachstehendes Beispiel zur Bekräftigung des Konstanzprinzips des molekularen Drehungsvermögens entlehnt Tschugajew der Dissertation von Kischner:

		$(L)_d$	$(M)_d$
6)	Menthylamin	— 36.15	— 56.03
	Dimethylmenthylamin	— 58.66	—107.35
	Diäthylmenthylamin	—114.8	—242.2
	Dipropylmenthylamin	—130.75	—312.5
	Dibutylmenthylamin	—114.35	—305.3

Es ist ja aber auch hier $(L)_d$=—130.75 nicht $(L)_d$=—114.35 und $(M)_d$= =—312.5 nicht $(M)_d$=—305.3 gleich. Hier haben wir wieder ein Maximum $(M)_d$=—312.5, dem eine Abnahme folgt.

Einen ebensolchen Umschlag sehen wir auch in einem dritten von Tschugajew selbst angeführten Beispiel:

		$(L)_d$	$(M)_d$
7) Monochloressig	Ester des Menthols	— 73.86	—171.0
Dichloressig	» » »	— 63.56	—168.7
Trichloressig	» » »	— 59.05	—176.8

Als Umschlagspunkt dient das Minimum $(M)_d$=168.7. Eine ebensolche Erscheinung des Umschlags der Verhältnisse finden wir in einer ganzen Reihe von anderen Beispielen.

		$(M)_d$
8)	Diäthyläther der Diazetylweinsäure	29.58
	Diäthyläther der Diproponylweinsäure	88.16
	Diäthyläther der Dibutyrylweinsäure	10.38
9)	Methylamyläther	0.4
	Äthylamyläther	0.7
	Propylamyläther	1.17
	Butylamyläther	1.91
	Diamyläther	0.77
10)	Methyläther der Azetyläpfelsäure	39.8
	Äthyläther » »	42.5
	Äthylpropyläther » »	45.8
	Äthylbutyläther » »	42.0
11)	Äthylamyläther der Oxalsäure	11.78
	» » Malonsäure	8.09
	» » Bernsteinsäure	9.71
	» » Glutarsäure	9.54
12)	Dimethyläther der Brombernsteinsäure	115.15
	Diäthyläther » »	103.63
	Dipropyläther » »	106.9
13)	Methyläther der Phenylchloressigsäure	198.43
	Äthyläther » »	49.9
	Propyläther » »	50.87
	Amyläther » »	56.08
14)	Dimethyläther der Nitroäpfelsäure	— 68.33
	Diäthyläther » »	— 71.8
	Dipropyläther » »	— 67.5
15)	Isobutyläther der Azetylmilchsäure	— 91.2
	» » Propylmilchsäure	— 95.8
	» » Butylmilchsäure	— 82.7

All das Obige zusammenfassend, kommen wir zu nachstehenden Schlüssen:

1. Die in den von allen obenerwähnten Forschern zur Ermittelung bestimmter Gesetzmässigkeiten in der Abhängigkeit des Drehungswertes von der chemischen Struktur der Substanz aufgestellten Hypothesen und Regeln vorkommende Widersprüche sind so wesentlich, dass diese Regeln wohl kaum als allgemeingültig für sämtliche homologe Reihen der optisch aktiven Substanzen betrachtet werden dürfen.

2. Der Wert des $(M)_d$ in homologen Reihen optisch aktiver Substanzen nimmt mit dem Anwachsen des Molekulargewichts des Substituenten nicht immer k o n t i n u i e r l i c h zu oder ab. Meistens führen diese Ab- und Zunahme zu einem gewissen Maximum oder Minimum.

3. Dieses Maximum oder Minimum bleibt in den nächsten Gliedern der homologen Reihe durchaus nicht als konstanter Wert: es folgen dem Maximum oder dem Minimum weitere Ab- oder Zunahme des Wertes des $(M)_d$. Mit anderen Worten: In der Veränderung des Wertes $(M)_d$ bei dieser oder jener homologen Reihe optisch aktiver Substanzen tritt die Erscheinung des Umschlags der Verhältnisse am häufigsten ein.

4. Es kommen Fälle vor, wo in einer homologen Reihe die Veränderung des Wertes $(M)_d$ zwei Umschlagspunkte (zwei Maxima, zwei Minima, ein Maximum und ein Minimum) herbeiführt. Es kommt auf diese Weise der Verlauf der Veränderung durch eine wellenförmige Kurve zum Ausdruck.

5. Das Prinzip des Umschlags der Verhältnisse in der Veränderung des $(M)_d$-Wertes der Glieder homologer Reihen optisch aktiver Substanzen ist keine zufällige Erscheinung. Sie ist eines der Elemente eines ganzen Systems von periodischen Wechselbeziehungen auf dem Gebiete organischer und anorganischer Abkömmlinge.

6. In gewissen homologen Reihen optisch aktiver Substanzen fehlt das Prinzip der Unterbrechung der Kontinuität. Dies liegt wohl daran, dass viele homologe Reihen optisch nur unvollständig untersucht sind: der Umschlagspunkt liegt möglicherweise bei einem der Glieder, die optisch noch nicht untersucht worden sind.

2. Veränderung des $(M)_d$-Wertes in Abhängigkeit von dem Sättigungsgrad der Gruppen

Die Erscheinung des Umschlags der Verhältnisse kommt vor in der Veränderung des $(M)_d$-Wertes in Abhängigkeit von dem Sättigungsgrad der Gruppen.

Walden ([8]) zeigt an einer ganzen Reihe von Beispielen, dass der $(M)_d$-Wert mit dem Übergang von einer einfachen Bindung der Kohlenstoffatome zu einer doppelten zunimmt.

		$(M)_d$
N-ölsaurer Äther	$CH_3—CH_2—CH_2—COOC_5—H_{11}$	4.43
Krotonsaurer Äther	$CH_3CH=CHCOOC_5—H_{11}$	6.62
Bernsteinsaures Diamyl	$\begin{matrix} CH_2—COOC_5—H_{11} \\ \vert \\ CH_2—COOC_5—H_{11} \end{matrix}$	9.71
Fumarsaures Diamyl	$\begin{matrix} CH—COOC_5—H_{11} \\ \Vert \\ CH—CO—C_5H_{11} \end{matrix}$	15.17
Methylbernsteinsaures Diamyl	$\begin{matrix} CH_2—COOC_5—H_{11} \\ \vert \\ CH_3—CH—COOC_5—H_{11} \end{matrix}$	9.99
Mesakonsaures Diamyl	$\begin{matrix} CH—COOC_5—H_{11} \\ \Vert \\ CH_3C—COOC_5—H_{11} \end{matrix}$	16.0

Der Übergang von einer doppelten Bindung der Kohlenstoffatome zu einer dreifachen wird von Walden nur an einem Beispiel illustriert:

		$(M)_d$
Zimtsaures Amyl	$C_6H_5CH=CH—COOC_5—H_{11}$	16.36
Phenylpropylsaures Amyl	$C_6H_5C\equiv CCOOC_5—H_{11}$	12.0

Hier hat beim Übergang von der doppelten Bindung der Kohlenstoffatome zur dreifachen eine Abnahme des $(M)_d$-Wertes statt.

Es nimmt somit der $(M)_d$-Wert beim Übergang von einer einfachen Bindung zu einer doppelten zu, wogegen er beim Übergang von einer doppelten zu einer dreifachen Bindung abnimmt.

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ZITIERTE LITERATUR

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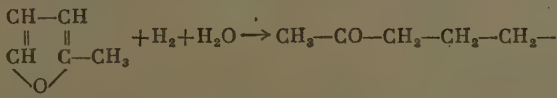
CHEMISTRY

ON THE MECHANISM OF THE FORMATION OF γ-ACETOPROPYL ALCOHOL DURING THE HYDROGENATION-HYDRATION OF α-METHYL-FURANE AND ON THE CONSECUTIVITY OF HYDROGENATION OF THE DOUBLE BONDS IN THE α-METHYL-FURANE

By K. TOPCHIEV

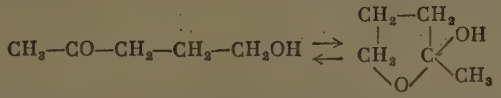
(Communicated by A. E. Favorskij, Member of the Academy, 27. III. 1938)

Topchiev and Pavlov (1) have shown that during the hydrogenation-hydration of α-methyl-furane the latter is converted with a good yield into γ-acetopropyl alcohol:



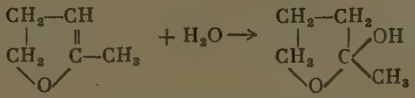
The mechanism of this process can be explained in the following way.

If Helferich's (2) views developed in his papers on the tautomerism of 1.4 and 1.5-oxyketones and oxyaldehydes are adopted, one can consider γ-acetopropyl alcohol as an instance of cyclo-oxo-tautomerism of a sugar type:



The results obtained by W. Herold (3) with the absorption spectra of compounds of this kind bear witness to the presence of such tautomerism.

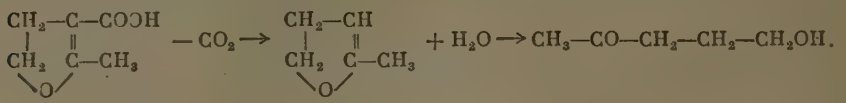
It is, therefore, undoubtedly that the hydration of methyl-dihydrofurane formed in the course of catalytic hydrogenation of methyl-furane in an aqueous solution goes through the addition of water elements to the unsaturated α, β-carbon bond with the formation of a cyclic form of acetopropyl alcohol.



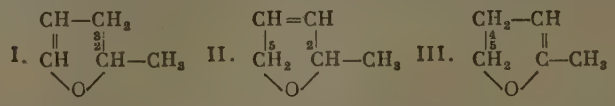
On the other hand, the very fact of formation of acetopropyl alcohol from methyl-furane under these conditions compared with the observations of Marshall and Perkins (4) on the attitude of methyl-dihydrofurane

towards water suggests an important conclusion on the sequence of the hydrogenation of double linkages in a molecule of α-methyl-furane.

Methyl-dihydrofurane (4.5) was obtained by Perkin by decarboxylation of methyl-dihydropenton-carboxylic acid prepared by him by means of a synthesis in which aceto-acetic ether had been involved. He has shown also that the methyl-dihydrofurane formed is converted in presence of water into acetopropyl alcohol:



Thus, out of the three isomeric methyl-dihydrofuranes (2.3; 2.5; 4.5) that could be formed during the catalytic hydrogenation of methyl-furane:



the first (2.3) during the hydrolysis should have given oxyvaleric aldehyde $CH_3—CHOH—CH_2—CH_2—CHO$; the second (2.5) having a tetrahydrofurane grouping in that part of the molecule which undergoes hydrolysis should not react with water at all, as analogous to the attitude of tetrahydromethyl-furane towards water, studied by Erenthal ([5]); and only methyldihydrofurane of the form III (4.5) can give acetopropyl alcohol.

The above statements in addition to the fact of formation of acetopropyl alcohol with good yield during the hydrogenation-hydration of methylfurane allow to draw a conclusion that the consecutivity of hydrogenation of the double bonds in α-methyl-furane in these conditions is such that a double bond between the unsubstituted carbon atoms is hydrogenated first of all.

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GEOCHEMISTRY

ON THE NEUTRONS OF ROCKS

By G. V. GORŠKOV, N. M. LJATKOVSKAYA, A. G. GRAMMAKOV and V. S. ŽADIN

(Communicated by V. I. Vernadsky, Member of the Academy, 27. III. 1938)

It is well known that a series of light elements, as heavy hydrogen, lithium, beryllium, boron, nitrogen, fluorine, sodium, magnesium, aluminium, phosphorus, chlorine and others, emit neutrons being bombarded by α-rays. Among the light elements, aluminium, nitrogen and magnesium* only have a notable expansion in the earth's crust ([1], [2]). It is known too that radioactive elements are in varying quantities dissipated throughout the earth's crust. One has therefore to suppose that the α-rays of radioactive substances while bombarding the nuclei of these elements will create neutrons in the earth's crust. In heavy water and beryllium neutrons will originate also under the influence of γ-rays of radioactive substances ([4]). Rough calculations show that neutron radiation of the earth's crust will be very small. Since adequate data on the emission of neutrons by the above elements under the action of α-rays of various radioactive substances as well as on the neutron velocities and the true absorption of neutrons having different energies are unavailable, we are for the time being unable to estimate the effect of neutron radiation of the earth's crust with sufficient certainty theoretically.

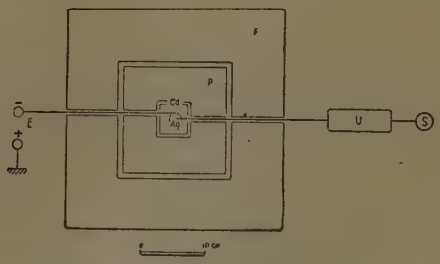
The present work had for object to experimentally evaluate the neutron radiation of rocks. When devising the arrangement for the experiment, it was necessary to secure optimal conditions for neutron detection in the presence of a rather intensive γ-radiation. The Vynn-Williams counter answers these conditions. Taking however into consideration the complexity of this apparatus and the long time required for its construction, we resorted to a silver counter which was a radiated target at the same time.

In order to reduce the background of gamma-rays the silver counter was placed into a cast-iron cuirass with walls 8 cm thick. A scheme of the arrangement is shown on Fig. 1, where *E* is high voltage (dry battery of 1500 V), *Ag*—silver counter 6 cm long (the active length), with an outer diameter of 1.8 cm and walls 0.15 cm thick, *Cd*—cadmium screen with walls 0.6 mm thick (or an iron screen of equivalent thickness), *P*—paraffin wax screen 5.5 cm thick, *F*—cast-iron screen 8 cm thick, *U*—Jonson amplifier, *S*—mechanical counter with maximal counting rate of about 1700 impulses per minute.

* According to Clark([3]) the average composition of igneous rocks forming the principal part of the earth's crust is as follows: SiO_2—59.12%, Al_2O_3—15.34%, Fe_2O_3—3.08%, FeO—3.8%, MgO—3.49%, CaO—5.08%, Na_2O—3.84%, K_2O—3.13%, TiO_2—1.05%, P_2O_5—0.30% and others —1.77%.

The cast-iron shield had the form of a hollow prism and could be shut by cast-iron plates, 6 cm thick, from the front and back sides.

The investigation of the neutron radiation intensity embraced three sets of measurements. The first one was preliminary and consisted of systematical measurements of the natural background of the counter. The counter was surrounded by a cast-iron cuirass and paraffin screen, besides iron and cadmium screens were applied which alternate with each another every 30 minutes. The neutrons stopped in paraffin and having passed through the iron screen hit the silver counter and activated it, the absorption of slow neutrons in iron being relatively small. In the presence of a cadmium screen the whole group



C of slow neutrons was absorbed by the latter and as group *C* composed under our experimental conditions the main part of all the slow neutrons, the activation of the silver counter in this case was small*. The activation with fast neutrons may generally be neglected as compared to the activation with slow neutrons, and therefore at first approximation it may be considered that the difference in the background will characterize the intensity of the neutron radiation.

The data obtained are shown in Table 1.

Table 1

Effect obtained with iron screen			Effect obtained with cadmium screen			Difference (effect per minute)
Total effect	For the period of observation	Effect per minute	Total effect	For the period of observation	Effect per minute	
10164	1075	9.45±0.09	9796	1061	9.23±0.09	0.2±0.2

The second series of measurements fully corresponded to the first one; but about 500 kg of uranium rock with 0.2% concentration U_3O_8 were placed under the counter arrangement, which exceeds the radioactivity of normal granites approximately by 200 times. The rock used had an increased content of uranium and phosphorus as compared to the normal one, however, no accurate analysis has been made as yet. The rock was arranged in the form of a pile: 100 cm long, 70 cm wide and 50 cm thick. The upper basement of the pile was 40 cm from the counter. The results obtained are tabulated below:

* According to the data obtained by Amaldi and Fermi[5] in the inner experiment, that is when the radiated target is all around enclosed by paraffin wax, the activation of silver by slow neutrons of the *C* group constitutes 84% of the total activation by slow neutrons.

Table 2

Effect obtained with iron screen	Effect obtained with cadmium screen	Difference (effect per minute)
$\frac{51\,092}{1892} = 27.4 \pm 0.12$	$\frac{45\,622}{1753} = 26.0 \pm 0.12$	1.4 ± 0.24

To check whether the emission of gamma-rays by iron is due or not to a possible content of small quantities of radioactive substances, a control experiment without iron was carried out.

The measurements were made in the presence of a radioactive rock, the cadmium screen being introduced into the apparatus every 30 minutes; during the 30 minute intervals the effect was measured without iron screen (the counter was surrounded with cast-iron and paraffin screens only).

The results obtained are given in Table 3; the last column shows the effect minus the absoption of γ-rays in cadmium screen computed by a mere exponential function.

Table 3

Effect obtained without cadmium	Effect obtained with cadmium	Difference (effect per minute)
$\frac{36\,678}{1407} = 26.0 \pm 0.13$	$\frac{30\,789}{1278} = 23.9 \pm 0.14$	1.5 ± 0.27

The table shows that the effect of the neutron radiation is in the limits of error equal to that measured with iron; there is no reason therefore to consider the effect obtained earlier as due mainly to contamination by iron.

Finally in the third series analogous measurements were carried out with an artificial radon+beryllium preparation with an energy of 0.3 millicurie (at the beginning of measurements). The ampulla was placed over the counter at a distance of 60 cm from its centre.

The data obtained, corrected for radon disintegration during the measurements, are shown in Table 4.

Table 4

Effect obtained with iron screen	Effect obtained with cadmium screen	Difference (effect per minute)
$\frac{69\,405}{577} = 120 \pm 0.5$	$\frac{63\,070}{551} = 114 \pm 0.5$	6 ± 1

During these measurements rock was present, so that the pure effect of the ampulla is 92.6 with the iron screen and 88 with the cadmium one.

The difference due to neutrons of the ampulla will be expressed by the number of 4.6 impulses per minute. These last measurements serve to check the sensitivity of our apparatus and make it possible to evaluate the neutron radiation of the rock subjected to measurement. In fact, using arti-

ficial preparation we registered 4.6 neutrons per minute and 88 gamma-quants per minute, that is one neutron per 19 gamma-quants.

When using rock, one neutron per 14 gamma-quants was registered.

Taking into consideration the geometry, it may be hence concluded that from the rock investigated approximately the same ratio of neutrons to gamma-quants as in the artificial preparation radon-beryllium has been obtained. If we assume that our apparatus registered an equal ratio of neutrons both from artificial preparation and rock, it may be approximately considered that per 2000 gamma-quants the rock emits one neutron*.

The intensity of the neutron radiation may be expressed in another way too, so as to facilitate the comparison of our results with those of other authors, which have observed neutrons in the atmosphere ([8]).

From the experimental results given above it is easy to calculate (assuming that our arrangement registers an equal ratio of neutrons both from the artificial radon+beryllium preparation, and from the natural material) that from a rock with a radioactivity of the order of $\frac{1 \cdot 10^{-2} \text{gRa}}{\text{g}}$ weshall have about 2 neutrons per hour referred to every square centimetre of the counter surface which agrees with Fünfer's results. The comparison presents, however, some difficulties, as we counted fast neutrons and Fünfer—slow ones.

Further experiments concerned with the study of neutron radiation of artificial object are in progress.

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* 1 millicurie Ra+Be emits every second $5.4 \cdot 10^7$ hard gamma-quants([6]) and $2.5 \cdot 10^4$ neutrons ([7]), whence the number given in the text is obtained.

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PETROGRAPHY

FLUORITE IN THE KUNGURIAN LIMESTONES AND DOLOMITES OF THE URAL-EMBA REGION

By V. P. BATURIN

(Communicated by V. I. Vernadsky, Member of the Academy, 28. III. 1938)

In studying the complex of chemical sediments in the zone of development of salt dome structures of the western Kazakhstan, the author has discovered fluorite in the carbonate rocks of the Kungurian. The rather peculiar mineral association in which fluorite was encountered, and the very fact of the discovery of this mineral in Kungurian deposits is of such a great interest that the author decided to make this preliminary communication before the whole of research work on the chemical sediments of salt domes has been completed.

Fluorite has been discovered in the limestones and dolomites of the Zamyatin dome. This dome is located at the bank of the Emba river, 180 km northward from the town of Aktyubinsk. The geological conditions of the salt dome area may be briefly described according to the data of V. E. Ruzhenzov, from his investigations made in 1929.

«In the central part of the dome the core (Kungurian) is projecting, not more than 100 m in diameter. In the north-eastern part the gypsums are overlapped by strong limestones. 100—150 m eastwards from the gypsums hills are located, composed of coarse sandstones of the Albian-Cenomanian, dipping 30—40° aside from the core. In the northern part of the dome, somewhat eastward from the core, a meridional thrust fault may be seen bringing black Aptian clays onto the surface».

Carbonate rocks have been taken from several small test-pits near by the gypsum bassets. A microscopic analysis has shown two layers: a reddisch-brown porous dolomite of arenaceous appearance, underlain by a light-grey compact limestone.

The methods of investigation of the material available were somewhat different from those generally used: aside from the examination of plan-parallel thin-sections the insoluble residue was isolated from the rock. This method made it possible to discover the fluorite, which was almost undiscernable in thin-sections. For the solution of the rock hydrochloric acid diluted 1 : 10 was used. A 50 g sample was treated by HCl in the cold for 24 hours. This being done, the large 8-litre jars, in which the solution had taken place, were poured full with water and the insoluble residue freed by elutriation from particles under 0.01. The dried insoluble residue was separated into the light and the heavy fraction in A. I. Moshev's centrifugal machine with Tule liquid; specific gravity 2.9. If the insoluble residue contained coarse grains it had to be screened first through a 0.25 mesh screen in order to facilitate the subsequent microscopic analysis.

The results of the isolation of insoluble residues from the Kungurian carbonate rocks of the Zamyatin dome have been rather unexpected. In four of the samples analysed the quantity of the isolated insoluble residue varied from 3 to 20%. At first sight there was nothing peculiar in it, since the rocks had an «arenaceous» appearance; it has been found, however, that the insoluble residue consists chiefly of celestine with an admixture of fluorite. Terrigenous material as quartz and feldspars is present in the limestone in quite negligible quantities, tenths and hundredths of a per cent. Thus, the light fraction is almost absent in the insoluble residues of these limestones. The heavy fraction, composed of a mixture of celestine and fluorite, as noted above, comprised above 99% of the insoluble residue. Since the fluorite content of the insoluble residue is about 2%, and the maximum of the latter constitutes about 20% of the whole mass of the rock, it is easy to calculate the value of the fluorite content in the limestone; it is at the most some tenths of a per cent.

The shape of the celestine grains is irregularly isometric, less frequently prismatic crystals are present as a combination of prismatic and pyramidal faces. The size of the celestine grains varies from several hundredths to two tenths of a millimeter; the celestine crystals contain frequently opaque inclusions chiefly of an argillaceous character. Fluorite occurs in cubes and irregular grains, the sizes of both varying within 0.02—0.05 mm. The skeletons of dissolution, formed most probably at the expense of processes of carbonate rock leaching, are of great interest. It is less probable that the fluorite has been subjected to corrosion during the treatment of the sample with diluted hydrochloric acid.

In separate laminae of celestine small cubes of fluorite are frequently enclosed. This observation is of great importance since it shows the sequence of precipitation of the minerals of the insoluble residue, namely, it attests the primary isolation of fluorite, followed immediately by the formation of celestine.

In the sedimentary rocks of the USSR fluorite is rather widely spread. Since 1808, when Fisher discovered this mineral near Vireya, many other findings of fluorite have been recorded chiefly in the territory of the Russian platform. All these findings, however, have been confined to the Carboniferous. Not before 1936 N. A. Uspensky discovered fluorite in paragenesis with celestine in Artinskian deposits of the eastern slope of the Urals. In deposits of other ages fluorite has been encountered in many other countries, as for instance in the Permian deposits fluorite is known chiefly in Central Europe. All the investigators who have encountered fluorite in different deposits were chiefly interested in the genesis of this mineral, and particularly in the sources of fluorine, a rather rare element in sedimentary rocks.

In a recent paper by G. I. Bushinsky the different viewpoints on the origin of fluorite are grouped as follows:

1) Formation of fluorite at the expense of fumarol and hydrothermal processes (A. P. Karpinsky, B. P. Krotov and others).

2) At the expense of concentration of fluorides disseminated in the sediments (R. Morrison, A. E. Fersman and others).

3) At the expense of active organisms (J. V. Samoylov, A. E. Fersman).

4) In the course of drying of relic basins (Mackie).

Without going into a criticism of all the theories, which have been scrutinized in the above mentioned paper by G. I. Bushinsky, we shall here only discuss the last interpretation as to the origin of fluorite, suggested by the well known english petrographer V. Mackie.

This interpretation seems to be attractive because many features (facial connection with gypsum, bandy structure, absence of organic residues,

etc.), which are an evidence in favour of a chemical origin, characterize the Kungurian carbonate rocks, as the author's observations in the Aktyubinsk region have shown.

The whole complex of the Kungurian deposits of the Ural-Emba region (salt, gypsum, silvite, etc.) is an evidence for the conditions of the drying basins, characterizing this geological epoch. Many works of great geologists are dealing with the history of the Permian time and for this reason there is no occasion of dwelling upon this theme in the present note.

It is only important for us that the beds of chlorides, sulphates and carbonates have been deposited in different sequences in different areas of the saline Permian basin. Towards the close of this impressive process of chemical sedimentation, the residual masses of water were enriched in rare elements.

In the table, in which the chemical composition of sea-water is given, the course of this process may be clearly seen. The first decade encloses elements of water, O and H, the second, third and fourth—the fundamental components of the salt composition—Cl, Na, Mg, S, Ca, K. The elements V and VI of the Br, C, Sr, B, F decade are of minor importance, since they are contained in 100th and 10 000th of a per cent.

Chemical Composition of Sea-Water* (Weight in %)

I	O H	85.80 10.67	VI	B F	$4.5 \cdot 10^{-4}$ $1 \cdot 10^{-4}$
II	Cl Na	2.00 1.07	VII	Si, Rb, N, Zn, Ba	10^{-5}
III	Mg	$1.4 \cdot 10^{-1}$	VIII	Li, I, Fe, As, P, Al, Cu, Mn	10^{-6}
			IX	Cs	10^{-7}
IV	S Ca K	$9 \cdot 10^{-2}$ $4.5 \cdot 10^{-2}$ $3.8 \cdot 10^{-2}$	XI	Ag	10^{-9}
			XII	Au	10^{-10}
V	Br C Sr	$6.5 \cdot 10^{-3}$ $3.5 \cdot 10^{-3}$ $1 \cdot 10^{-3}$	XIV	Ra	10^{-12}

Without considering the still rare elements, contained in sea-water in quantities less than 10^{-4}, let us turn to the characteristics of the V and VI decades, to which fluorine is referred. Through a number of genetic features the five elements of the V and VI decades may de subdivided into three groups: a) C, b) Br, c) Sr, B, F.

C a r b o n. The position of C is quite peculiar, since C is being supplied to the ocean not only from dry land with river waters, but also from the troposphere as CO_2. Though the question as to the chemical origin of certain types of finegrained limestones has not been finally solved, there is hardly any doubt, however, that in certain geological moments huge masses of calcium carbonate accumulated at the sea bottom, as a result of chemical precipitation. The chemical sedimentation of thick beds of carbonate rocks with a comparatively low concentration of C in the sea-water may be explained by the great reserves of C and the high cation content.

* Table taken from «Chemical Elemental Composition of Marine Organisms», A. P. Vinogradov, Pt. 1. The data are generalized beginning with the VII decade.

Bromine. Bromine belongs to the dispersed vadose elements; it forms mineral compounds very rarely and escapes petrographical observation.

Strontium, borone and fluorine. Aside from their similar ratio in sea-water they approach one another in their ability to form stable minerals. For this reason on the background of a precipitation of the chief components—rock salt, anhydrite, silvite, etc., we are justified in expecting an isolation just of the minerals Sr, B, F as chief «accessory minerals» of the salt-bearing formations.

The most widely spread accessory mineral is celestine $SrSO_4$. This may be explained first by a higher Sr content in the sea-water against B and F. Other factors, however, e. g. the solubility of the forming minerals etc., may possibly exercise their influence too. The celestine of the Permian deposits has been studied by B. V. Samoilov, B. P. Krotov and L. M. Miropolsky.

«The celestine,—says Miropolsky,—is a most characteristical mineral of the Permian deposits, and on account of its wide occurrence may be classed among other characteristical minerals of the Permian system, namely, gypsum, rock salt, anhydrite, copper compounds, etc.».

Our investigations show that celestine is rather wide spread in the Kungurian of the Ural-Emba region too. Aside from the limestones and dolomites described, celestine has been encountered in the gypsums of several domes—Karachungul, Zamyatino, etc.

The distribution of the boron minerals in the salt-bearing formation of the Ural-Emba region has a local character (Lake Inder). There is a possibility, however, of a more extensive occurrence. The geochemical investigations made by A. A. Varov showed B in the waters of several salt domes.

The third element of this group—fluorine, which is present in sea-water in quantities approaching Sr and B and forms with calcium a stable mineral species—fluorite, should also be crystallized in the course of metamorphism and of the process as a result of which the Permian basin becomes more saline. N. A. Uspensky is of the same opinion; he encountered celestine concretions with small inclusions of fluorite in the Lower Permian gypsums. The findings of fluorite in the Kungurian rocks of chemical origin in association with celestine confirm the correctness of our conception as to the features of genetic similarity of Sr, B and F in salt-bearing deposits. Our material is much more conclusive than that of Mackie, with regard to the chemical origin of fluorite. Mackie discovered fluorite in arenaceous continental sediments, whose sedimentation conditions it is hardly possible to retrace in detail; no mineral formations, whose chemical origin was probable and indubitable, have been encountered in immediate association with fluorite*. In the Ural-Emba region and in Urals fluorite has been encountered in carbonate rocks and gypsums, accompanied by celestine, a mineral formed also in result of chemogene processes—after a prolonged enrichment of the saline basins by strontium salts, parallel to which an increase of B and F concentration should always take place.

Since the fluorine content in sea-water is 10 time less than that of Sr and 4.5 times less than that of B, the fluorite occurrence in the salt-bearing formations should be correspondingly rarer as compared to celestine (and borates), which has also been the case in the samples studied by the author.

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* Baryte concretions have been discovered in an adjoining area of the same deposits.

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SOIL SCIENCE

ON THE RATE OF SOIL TILLAGE

By S. I. YARJEMSKY

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It must be confessed that to this time the very basic principle underlying the mechanics of soil tillage remain in many cases insufficiently known and it is this want of knowledge which appears to be responsible for a number of erroneous conceptions, in the light of which the relation of tillage speed to power indices is considered. As a rule, the resistance to deformation with tillage was incorrectly considered to be small and to depend but little on deformation rate; from the analytical angle, this dependence was essentially deduced from the equation of kinetic energy written for the case of the throwing aside of furrow slice particles (Prof. V. P. Goriatchkin); and this involved a parabolic form for the analytical expression of the interrelation of strain and velocity (Goriatchkin's formula). This equation, examined within the limits of the comparatively small velocities used in practice, has received extensive application and has yielded results meeting experimental requirements up to 2—2.5 m/sec.

However, a study of the mechanical principles of the main process of disintegration (pressure—compression) and of the respective compression dependence (1) brings out most forcibly the sufficiently strong influence of the velocity of deformation upon the magnitude of the latter in diagrams of pressure—compression. The effect in question will, however, be liable to vary for a number of concrete cases under consideration, depending on changing moisture content.

On the other hand, notwithstanding the accepted view, it has been well ascertained even by very preliminary examination of the fundamental mechanisms at work here that resistance to deformation and stress plays a very considerable rôle in the process.

Such considerations permit us to advance a new form of dependence, viz. velocity—tractive force, based upon the true mechanical essence of the deformation process and properly framed with respect to the compressional dependence of deformation on pressure.

According to Terzaghi, the dependence between pressure, p, and deformation, c, may be represented as follows:

$$\varepsilon = -\frac{1}{A} \ln (p_s + p_c) - \beta (p_s + p_c) + c, \qquad (1)$$

where p_c, A, β and c are constant, while ε is the deformation of compression, which is negative.

Omitting insignificant magnitude β and denoting

$$p_s + p_c = p;$$

we have

$$\varepsilon = -\frac{1}{A} \ln p + c; \tag{2}$$

$$p = e^{A(c-\varepsilon)},$$

where, according to Terzaghi (Erdbaumechanik, pp. 11 and 12),—$\varepsilon = f(v)$.

Denoting by $e^{Ac} = p_0$,

$$p = p_0 e^{f(v)}, \tag{3}$$

i. e. we may say on mechanical grounds that such a form of function should be regarded as the most probable functional dependence: strain—velocity.

This form of function finds itself at present a far-reaching application in studies of disperse bodies, such as soil. Indeed, such an expression characterizes with sufficient clearness the course of the processes from a mechanical point of view; wherefore the coefficient of such an equation, as will be explained below, should also have a fairly distinct physico-mechanical and technological significance.

It should also be noted that the dependence of stress, p, needed for the destruction of specimen (according to Atterberg), upon moisture w was represented by Christensen(2) in a similar and most convenient form:

$$p = p_0 e^{-kw}.$$

We may therefore infer from the above that the simplest form of dependence stress—velocity as advanced by the writer will have the appearance:

$$P = e^{av+b} \tag{4}$$

or

$$P = P_0 e^{a(v-v_0)}. \tag{5}$$

Using logarithmical paper and reducing equation (4) to logarithmical form, we shall readily see that the form proposed by us may find evidential support in the results of the latest experimental work of the Agricultural College, Alabama, USA(3), being as yet the only material satisfying the necessary conditions, such as embracing the full range of velocities with rates up to 4.5 m/sec. (10 miles per hour); making use of fairly accurate experimental equipment on trenches; performed on sufficiently representative USA soils with the use of several gang mouldboard ploughs.

Figs 1 and 2 illustrate the results of experiments made under the conditions just mentioned with the following types of soil: Norfolk sand, Davidson loam, Decatur clay.

The functional dependence has been concretely established by us for each type of soil mentioned as follows:

$$\left.\begin{array}{ll} \text{Norfolk sand} \ldots\ldots & P = P_0 e^{0.265(v-v_0)} \\ \text{Davidson loam} \ldots\ldots & P = P_0 e^{0.284(v-v_0)} \\ \text{Decatur clay} \ldots\ldots & P = P_0 e^{0.310(v-v_0)} \end{array}\right\} \tag{6}$$

In equation (4) coefficient a ought also to depend on soil texture as well as on moisture—a question requiring further experimental study.

Resorting to Fourrier's homogeneity theorem, we note from equation (4)

$$av = \text{Inv.}$$

It should be noted that the dimensions of a depend on those of velocity and consequently are numerically conditioned by the choice of the system of units. In deducing equations (6) velocity is expressed in m/sec.

The parabolic form of dependence

$$P = a + bv + cv^2, \tag{7}$$

suggested by Randolf and Reed is, as shown by the substitution, a practically convenient experimental formula for Davidson loam and Norfolk sand. However, it appears fairly difficult to give a precise characteristic to the coefficient of this equation from the stand point of mechanico-technological principles of the process and thus to apply it also to Decatur clay.

The well-known formula evolved by V. P. Goriatchkin

$$P = fg + kab + \varepsilon abv^2 \tag{8}$$

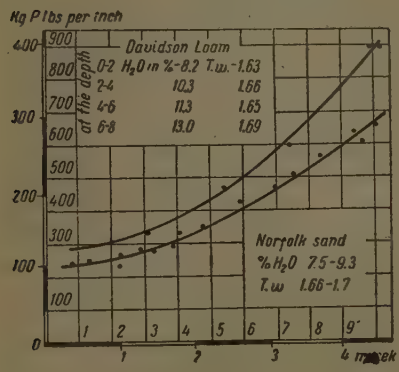

Fig. 1.

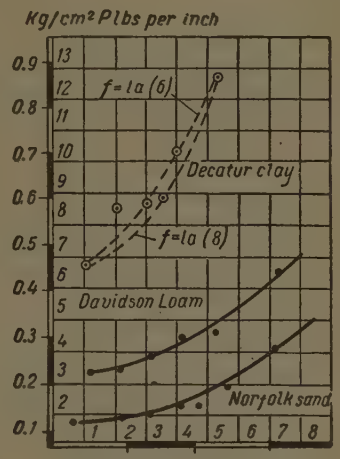

Fig. 2.

should be considered as a particular case of (7). When resorting to it, coefficients f, k and ε, as shown by analysis and experimental evidence, fail to have a definite mechanical significance.

On the other hand, the establishment of a parabolic dependence and the application of formula (7) in respect of Decatur clay proved practically rather difficult, which was noticed by Randolf and Reed[3]. Formula (8) is also difficult to be used for the case in hand. In Fig. 2 the results of experiments conducted by Randolf and Reed are plotted for comparison and concomitantly curves are drawn through the two extreme points, in accordance with equations (6) and (8).

Our treatment of experimental material for Norfolk sand and Davidson loam according to formulae (6) and (8) points to agreement of results with speeds commonly used in practice, while it indicates divergence at higher speeds; stress P determined according to formula (6) being, as a rule, somewhat higher than when determined according to formula (8).

Furthermore, the coefficient ε, for instance, determined separately (Davidson loam and Norfolk sand) from the equations of kinetic energy for the furrow slice thrown away, has a value which is several times smaller than that observed practically [according to formula (8)]. This fact fully agrees with the assumption that resistance to deformation and consequent change of velocity play a considerably greater part in the power balance than was hitherto supposed.

The part taken by energy expended in deformation by far exceeds the part taken by the energy of particles thrown away and the functional dependence between P and v should be built up not according to the equation of the kinetic energy of the particles thrown away, but after the compression curve which is apt to define the nature of the process in a much better way.

The form of dependence suggested by us

$$P = P_0 e^{a(v - v_0)}$$

is at the same time practically very convenient and may be used in logarithmical form with logarithmical paper:

$$\ln P = \ln P_0 + a(v - v_0).$$

The above stated discussion of one of the most vital present-day problems connected with the possibility of passing over to higher tillage speeds is based on a consideration of the mechanical characteristics of the soil deformation process, soil being here the material under treatment.

The agreement of the results of recent experimental studies dealing with higher speeds, with the application of our formula, emphasizes the great importance of the discussion of such problems in the investigation of mechanical working processes and put forward a new form of functional dependence sufficiently convenient for practical application, which in its structure is based on a close analysis of the very essence of the process involved. The work has to be corroborated by additional experimental evidence, defining more specifically the numerical values of P_0 and a both for different types of soil coming under the fundamental classification and for different texture and moisture contents within a separate type.

The mechanics and the mechanical technology of soil is a science which was but recently brought into the sphere of activity of international conferences ([4]). But at present, along with the developments of General Soil Science, it is to afford a sound basis for a concrete examination of mechanical processes in the soil, these processes virtually lying at the bottom of quite a number of outstanding problems of National Economy.

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BIOLOGY

THE SURVIVAL OF A BROAD TAPEWORM'S PLEROCERCOIDS (*DIPHYLLOBOTHRIUM LATUM* L.) IN ARTIFICIAL MEDIA

By G. MARKOV

(Communicated by I. I. Schmalhausen, Member of the Academy, 8. IV. 1938)

The cultivation of parasitic helminths in vitro is a very important problem. The possibility to keep the worms for some time in artificial conditions opens some new prospects in the study of the physiology of helminths.

The knowledge of the helminths' physiology, its study answers urgent practical needs and is very important for theoretical purposes.

The literature on the subject is collected in Wardle's review (2). The works carried on were leaching definite purpose and yielded meagre results. Only from the year 1933 on a Canadian parasitologist R. A. Wardle is working intensively in that scope, yet his result do not seem to be promising either. The plerocercoids of a broad tapeworm in the saline media at 38° C had lived in Wardle's experiments for 13 days (1). The larvae of tetrahynchid *Nylelina surmenicola* Okada had lived at +18—24° C no more that for 456 hours (2). The adult *Moniezia expansa* Blanchard at +38° C had lived 72 hours only (3).

According to various reasons, I chose the plerocercoids of the broad tapeworm as an object for cultivation in vitro. The cultures of 1900 plerocercoids were kept in aerobic oligoseptic conditions at 8—15° C in the dark; all media were prepared with the Ringer-Locke saline solution. The cultures were examined and the solution changed every 2—3 days.

The results of the most successful experiments are given in the table and in the chart. The second column of the table shows the ratio of the worms (in % to the initial number) which were living without any degeneration symptoms till the last day of the experiment. The third column gives the «critical day», i. e. the day, up to which 40% of the worms survived in the cultures. The fourth column shows the mean percentage of degenerating worms. It shows the percentage of degenerating worms (to the number of the worms present, but not to the initial number of them) registered at the repeated examination of the cultures.

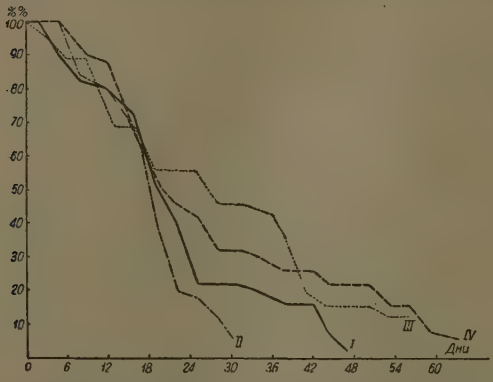
From the table and chart it may be seen that:

1. The media where the life of plerocercoids is the longest are: the fish broth (63 days) and the glucose with vitamins (56 days). The curve of decrease of the member of worms is in these experiments at its smoothest.

2. In the glucose with vitamins the worms live almost twice as long (56 days) as in the glucose without vitamins (30 days).

Media	Lifetime in days	Percentage of worms survived till the last day	The «critical day»	Average % of degeneration
Ringer-Locke	47	2	22	29
Glucose	30	6	17	24
Glucose + vitamins	56	13	37	19
Pepton	48	10	25	16
Fish broth	63	6	26	12
Fis broth gel + pepton + glucose	41	13	31	16
Pepton + glucose	46	2	31	19
Fish broth + glucose	36	20	31	9
Fish broth + pepton + glucose	37	8	19	20

3. The «protectional» influence of the nutritive media in comparison to Ringer-Locke is manifested: a) in a lower degeneration rate (29% in Ringer-Locke and 18% in other media), b) in a higher ratio of the worms which live till the last day of the experiment (2 and 10% respectively).



The daily numbers of the plerocercoids in per cent to their initial numbers: in Ringer-Locke (—*I*); in the glucose (—.—.—. *II*); in the glucose with vitamins (.....*III*); in the fish broth (— — — — *IV*).

4. The glucose solution proved to be the most unfavourable medium.

5. The plerocercoids do not live longer in combined media than in simple ones with only one exception (glucose).

6. The individual resistance of the plerocercoids to unfavourable cultural conditions is different. This has been pointed out by Wardle for *Nybelina* [2] too. Thus, 40% of the plerocercoids in my experiments were living 27 days on the average, but till the 46th day only 9% of the worms survived. Young plerocercoids (1—2 mm) lived half as long as the more adult plerocercoids.

7. The lifetime of plerocercoids observed in my experiments (46 days on the average) it quite sufficient for physiological experiments in artificial media to be made and much exceeds that observed in experiments of other workers.

In no case could I notice an increase of the volume, that is a growth of the plerocercoids. Notwithstanding I deem it very probable that the plerocercoids feed on artificial nutritive media and the length of their life in vitro might be prolonged still more. The greatest difficulty lies in finding out the conditions favourable for the growth of the worms.

The Peterhof Biological Institute
of Leningrad State University.

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MICROBIOLOGY

EXPERIMENTAL CHANGE IN THE PHYSIOLOGICAL PROPERTIES OF YEAST

By V. I. KUDRYAVTZEV

(Communicated by A. A. Richter, Member of the Academy, 1. IV. 1938)

The problem of experimental modification of microorganisms has its most interesting and important point in the question of how to produce new forms, with new fermentative propertiess.

Such instances are known in the literature on the subject (Effront, 1898, Hartmann, 1903, Massini, 1906, and others), but they are single cases. To many they seem not very convincing, and that is why they have not as yet attracted the attention they merit. If we could succeed in mastering this process fully, in understanding its essence, then we would have a powerful factor for changing the nature of the living organism.

We began working in this direction as early as in 1932. As experimental organisms we chose yeasts, since this group of microbes is most suitable for a study of variability problems. They have been studied more thoroughly than other microorganisms. Their size and more definite form permit better control of the experiments. Furthermore, since the end of the past century (Hansen, Klocker) it has been and is still considered that their fermentative properties are most constant. That is why any change in these properties should be especially convincing.

The experiments described below we started being convinced that the complication of the fermentative complex in various species of yeast belonging to the same genus (in other words, their ability to ferment an ever greater diversity of sugars) was growing in a definite sequence during the process of evolution, and that this complication process could be reproduced and continued under experimental conditions.

This conclusion was drawn partly as a result of a systematic study of yeast flora in various habitats.

A comparative morphological and physiological study of various species of yeast made it possible (at least with regard to certain species) to trace, hypothetically, the line of the gradual complication of the fermentative complex connected with an expansion in their habitat. Thus, for instance, it proved possible to conclude that *Saccharomyces paradoxus* (Batsch) and *Sacch. cerevisiae* var. *ellipsoideus* (Dekker) were two most closely related species of the genus *Saccharomyces*.

From an analysis of the difference between these two species* it was further

* A detailed analysis will be given in another notice.

assumed that the cultivated form of *Saccharomyces cerevisiae*, widely used by man in the fermentation industry, is historically a later species originating from the older wild species of the *Saccharomyces paradoxus* type, and having acquired the power to ferment a new sugar, viz. the maltose.

The latter could rightly be called «cultivated» sugar for it appeared in large quantities since man learned to make sugar out of starch and introduced this into practice first in the preparation of kvass (rye beer) and beer, and later in the production of alcohol. Under natural conditions it is found in insignificant quantities.

We hypothetically associated with this practical activity of man the appearance of a new species of *Saccharomyces (Sacch. cerevisiae)*, endowed

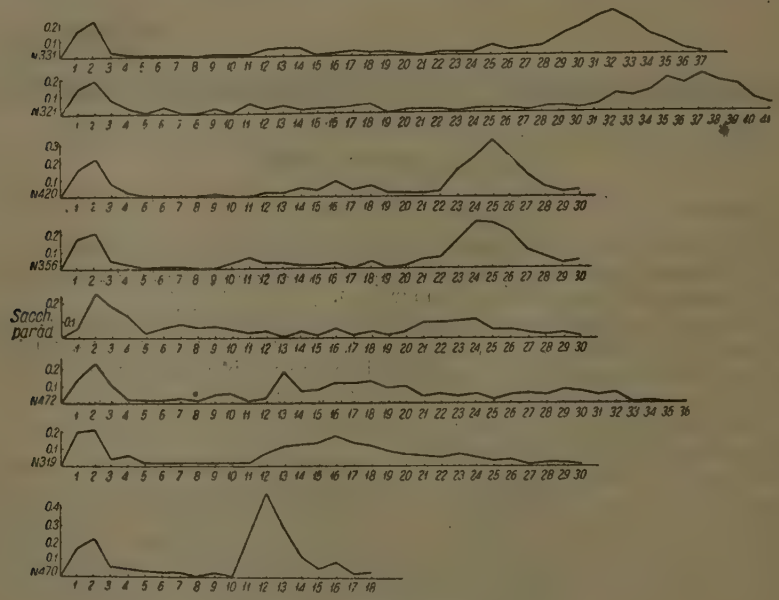

Fig. 1. Sacch. paradoxus.

with the power to ferment a new sugar, and afterwards unconsciously selected by man.

Using this working hypothesis, we decided to make an attempt to reproduce under experimental conditions that which had occurred at some time in the past—the case of a new formation of the maltose ferment.

The method of the experiment was suggested by the above-mentioned considerations.

As the nutritive medium for the development of *Sacch. paradoxus* was used strong (14° Ball.) unfermented wort, rich in nitrogenous substances and containing about 1—1½% glucose, fructose and saccharose, and about 8—9% maltose.

All cultures of the *Sacch. paradoxus* strains were isolated from one cell. The flasks with wort (50 cm³ in each), infected by these cultures, were sterilely closed with rubber stoppers provided with special locks which made it possible to rather accurately calculate the loss in carbon dioxide through fermentation. The flasks were weighed daily at a definite time. From the daily loss of carbon dioxide we judged of the fermentation process produced

by the test strains in the sugars of the wort and thus kept it under observation during a long period of time.

As a control we used yeast *Sacch. cerevisiae* XII, with which flasks with the same wort were infected separately under the same conditions. Numerous experiments with long fermentation of the wort by strains of the *Sacch. paradoxus* have evidenced that the latter at first use up only the usual fermentable sugars of the wort, i. e. glucose, fructose and saccharose. After this, fermentation ceased. The maltose remained unaffected. During this period it could be seen that the yeast not only ceased multiplication, but that under the influence of the products of metabolism and apparently as a result of carbohydrate starvation, part of the cells began dying. Further observation of the flasks attended by their daily weighing disclosed a renewed multiplication of several cells in the experimental cultures, which was accompanied by a repeated fermentation of the wort during which the maltose disappeared too. Peculiar two-peaked curves were thus obtained for the fermentation of the wort (Fig. 1).

In contradistinction to this, fermentation in the control flasks infected with *Sacch. cerevisiae* XII went on according to the type of one-peak curve (Fig. 2). This type points to a simultaneous fermentation of all the sugars of the wort, including the maltose.

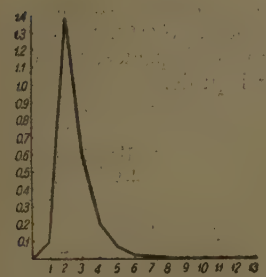

Fig. 2. Sacch. cerevisiae XII.

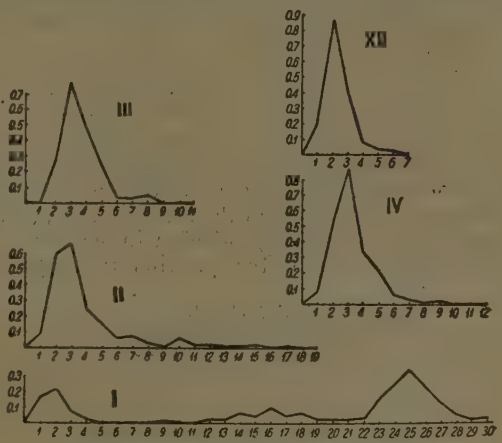

Fig. 3. Sacch. paradoxus № 420.

If we now transplant the cultures from the flasks with *Sacch. paradoxus* after the secondary fermentation into fresh wort and repeat the experiments of transplanting still further, then the curves of fermentation of the wort for the same duration of the experiments prove to be one-peaked, that is, they are similar in type to the controls (*Sacch. cerevisiae* XII), although the degree of fermentability of the wort is less (Fig. 3).

What has taken place in the flasks infected with strains of *Sacch. paradoxus?*

As shown above, each strain was isolated from a sole cell. Well controlled experimental conditions eliminated the possible infection of the flasks from without. Moreover, irrespective of the rate of the experiments, the results obtained were, on the whole, identical, differing only in details.

An examination of the precipitates of the yeast in the flasks after secondary fermentation of the wort showed that in the originally homogeneous cultures of *Sacch. paradoxus* there appeared new, morphologically similar forms of yeast possessing, however, a new peculiarity—that of fermenting pure maltose in a Hansen nutritive medium. Alongside of the new forms in the

cultures there always occur physiologically unchanged original forms of *Sacch. paradoxus.*

Analyses of many repeated experiments with fermentation of wort by the strains of *Sacch. paradoxus* enable us to draw the following conclusions:

1. All of the investigated (14) strains of *Sacch. paradoxus* when remaining for a more or less prolonged period (in individual cases up to 50 days) in an unfermented wort, were always giving origin to a new form of yeast endowed with the power of fermenting maltose. The ability to ferment other sugars (glucose, fructose, mannose, galactose, saccharose and partially raffinose) remained unchanged as compared with the initial form.

2. The new form of *Sacch. paradoxus* originates in the wort within the period following the termination of the initial fermentation of the wort, that is, after having used up the usually fermentable sugars of the wort (glucose, fructose and saccharose). Only individual cell organisms of the culture give rise to a new «maltose» form. The rest of cells of the original culture either die or remain at rest, being unable to develop at the expense of the maltose of the wort.

3. Not every newly originated «maltose» form of *Sacch. paradoxus* is valuable. Cases have been observed when this form died after three—four transplantations to fresh wort. In other cases the acquired ability of the yeast was maintained throughout the experiments.

4. According to its fermentative and morphological properties the maltose form of *Sacch. paradoxus* is similar to the cultivated species *Sacch. cerevisiae.* However, it differs from the latter in that, just as the initial species, it cannot form more than 6—7% alcohol in sugar-containing liquids.

Investigations on the question are in progress.

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HYDROBIOLOGY

THE VELOCITY OF THE MOVEMENTS OF CERTAIN PLANKTERS AND ITS DEPENDENCE UPON THE EXTERNAL CONDITIONS

By V. S. LUKJANOVA

(Communicated by S. A. Zernov, Member of the Academy, 2. IV. 1938)

In the preceding paper upon the problem of the influence of temperature and salinity on the velocity of the movement of fresh-water plankton organisms (*Daphnia magna* and *Cyclops* sp.), the author of this article had been able to show that within the limits between 15 and 30° the velocity of the organisms grows continuously with temperature and salt concentration of the water from 0 to 2.5‰ for the *Cyclops* and to 1.5—2‰ for *Daphnia*. Beyond this limit a negative effect of salinity is observed and if $S=10‰$ the velocity of the movement becomes considerably slower than under normal conditions ($S‰=0$).

The present investigation in some way continues the above work, yet on a more extensive material, both fresh-water and marine plankton organisms being the subject of the observation. In future similar observations will be carried on fish and sea mammalia. On the whole, the problem of the study of the influence of temperature and salinity on the velocity of the movement of all the water animals is particularly important and interesting, because it will allow us to learn the optima, maxima and minima of temperature and salinity, as well as the velocities of movement developed under these conditions by different species, especially by fish. Consequently we shall be able to calculate, at least approximately, the velocity of the migration of a school of fish under various hydrological conditions.

My methods were the following:

Water was poured into a glass dish 9—10 cm long, 7 mm wide and 7 mm high; the experimental animal was put therein. On the same level as the upper edge of the dish I put a slip of coordinate paper which was used as a scale for the record of the way passed by the organism. The time was recorded by means of a stop-watch. The glass vessel was put into a water bath heated to the necessary degree. The salinity wanted was created by adding salts, which were obtained of sea-water by means of evaporation.

1. Results of Experiments for Determination of Influence of Salinity on Velocity of Black Sea *Copepoda*. The graphs obtained for *Centropages kröyeri* Giesbrecht and *Paracalanus parvus* (Claus) are represented in Fig. 1 (*a*, *b*). The salinities were plotted along the axis of the abscissae, and the velocities, expressed in percentage of the velocity shown by the organism under the salinity of 18‰, along the axis of coordinates. As may be seen from the curves obtained,

the velocity of the movement gradually grows with salinity, from 18‰ to 25‰, while beyond this limit a negative effect of the salinity factor on the movement velocity is manifested. Curiously enough, we face here secondary maxima which arise both along the increase of salt concentration (32.5‰) and along its decrease (12—12.5‰). They are possibly due to the last convulsive movements of the animals which sometimes exceed in their velocity even the maxima obtained under 25‰ (*Centropages kröyeri*). Then comes the state

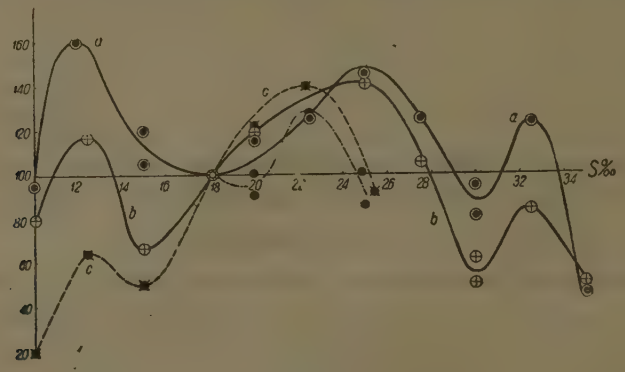

Fig. 1.

of torpor and rapid death of the animal. Under 10‰ the organisms made only 2—3 movements (leaps), then became perfectly motionless and no external impulses (blowing or touching with a stick) could make them move. When put back into normal water many of them came to life rather quickly, while others did not move at all. One can upon the whole conclude from my observations that the sea plankton organisms are more affected by light decrease of salinity than by its considerable increase.

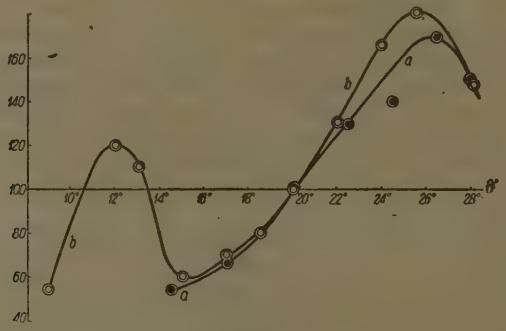

Fig. 2.

Besides we can see from the first two curves that the *Centropages kröyeri* responds to the change in the salinity of water much stronger than *Paracalanus parvus*. This may be accounted for by the fact that *Paracalanus parvus*, according to the work of Nikitin (1), is an eurithermal organism, which is able to stand larger variations of temperature. As all the eurithermal organisms are at the same time eurigalline (2) it becomes clear, why *Paracalanus parvus* gives a more smooth curve than *Centropages kröyeri*, which is a rather stenothermal and stenogalline organism.

In Fig. 1 we also see the curves representing the influence of the same factor (salinity) on the velocity of movement of the crab larvae (*c*) and polychaeta (*d*). For these two species the upper limit of velocity is 2.5‰ less, i. e. 22.5‰. In this case we again observe a second maximum under 12.5‰.

2. Effect of Temperature. The same material has been subjected to experiment, viz. *Centropages kröyeri* and *Paracalanus parvus*. The most complete picture was obtained for *Centropages kröyeri* (Fig. 2, *a*). As in the case of fresh-water organisms, the velocity proved to increase with temperature beginning from 15°; the maximum of velocity lies at 25.5° for *Centropages kröyeri* and at 36.5° for *Paracalanus parvus* (Fig. 2, *b*). In Fig. 2, *a* we also find a second maximum observed at the temperature of 12°. This is the temperature which, according to observations of Nikitin (1), is the limit under which we do not find the *Centropages kröyeri* in the sea. As to *Paracalanus parvus*, its lowest limit is 7—8° (1). From the above we can say beforehand that the second maximum (in the direction of the decrease of temperature) for the *Paracalanus parvus* should lie near 8°. The negative effect of the rise of temperature beyond 26° observed on both organisms perfectly agrees with the immediate observations made in the sea. Namely, by the end of August 1937 the surface of the Black Sea had an exceedingly high temperature, +28°, and just on these days our plankton tests came quite empty. Obviously this temperature badly affects plankton organisms, which descend therefore into deeper layers having a lower temperature.

From the data I obtained for fresh-water and sea plankton organisms, one can see that the upper limit of temperature is a little higher for the fresh-water organisms than for the marine ones. The velocity of the movements of *Daphnia* and *Cyclops* was still increasing at 30°, whereas that of the marine organisms was falling already at 28°. One had to expect this since the respective fresh-water reservoirs are heated much stronger and the organisms which live in are accustomed to higher temperatures.

Black Sea Hydrophysical Station.

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LIMNOLOGY

CAROTINOIDS IN LACUSTRINE SILTS

By S. MURAVEISKY and I. CHERTOK

(Communicated by S. A. Zernov, Member of the Academy, 2. IV. 1938)

In 1935 Baudisch and Euler first discovered carotinoids in lacustrine deposits. This was also the subject of a communication presented by Baudisch at the XV International Physiological Congress in Leningrad in 1935.

These authors have studied the deposits of two lakes: a lake from a swampy region in the environs of Stokholm and lake Belensee. The deposits of the first lake refer to algal gittia. Directly under the meadow soil there was a half-meter thick layer of peat, followed in some cases by a layer of calcareous gittia, and, in other cases, of red-coloured gittia, under which in all the cases there was a layer of olive green-coloured gittia. The thickness of the layers of gittia varied from 30 to 40 cm. The deposits of lake Belensee belong to coarsely detrital gittia. As extractors, sulphuric ether, acetone, carbon disulphide, alcohol and pyridine were used. Spectroscopic analysis has revealed that in peat newly taken out, in calcareous gittia and in coarsely detrital gittia xanthophyll is contained, while red and green algal gittia contain carotin. The amount of xanthophyll determined after the gravimetric method in calcareous gittia was found to be 34 mg per 1 g of organic matter of gittia, that of carotin (spectrophotometric determination), 50 γ per 1 g of raw red gittia and 42 γ per 1 g of raw green gittia; when recalculated to 1 g of dry matter of red gittia, there is 0.5 mg of carotin. Baudisch (1935) advances the problem of the planctonic silt of lakes as possible source of carotin.

In 1936 the present authors studied the silt deposits of a number of lakes of the USSR with the aim to find carotinoids.

A study was made of specimens both in an air-dry condition and freshly taken. The latter were treated immediately after the sample has been taken out. The samples ($^1/_{40}$ m^2) were taken from the bottom of the lakes with the aid of Petersen's dredger. The dry specimens were reduced to a finely powdered condition in a porcelain mortar, the wet ones were ground with twice or three times as great a quantity of anhydrous sodium sulphate until a homogeneous grey-coloured powdery mixture was obtained. The extraction was carried on in bottles with ground-in stoppers at room temperatures in dark rooms, the bottles being intensively shaken during the operation. As extractors, ethyl alcohol (96° or absolute), petroleum ether, sulphuric ether, chlorophorm were used. An exhaustive extraction lasted from 2 to 13 days. Use was also made of a successive exhaustive extraction with a group of different solvents (extraction prolonged to 30 days). The extracts obtained were in some cases directly tested by qualitative reactions for the presence of carotinoids; in other

cases they were subjected to a farther treatment. The extracts were sucked out with a Bücker funnel and purified from mechanical admixtures by filtering through a dense cotton plug. Since the extracts often contained a certain amount of chlorophyll or its derivatives, the latter were removed after the methods of Wilstetter-Schertz, Fremi-Timiriazev, Kraus. In the presence of yellow-orange pigments their mixture was separated in a two-phase system alcohol—petroleum ether into oxycompounds (xanthophylls) and hydrocarbons (carotins). A qualitative determination of carotinoids was made by means of a reaction with antimony trichloride (after Carr and Price) and spectroscopically; a quantitative determination was made colorimetrically, after Wilstetter-Stoll. The error of this method after Schertz (1923) amounts to 17%; after Delano and Dick (1933), to 11—38%.

On account of the imperfection of the method of Wilstetter-Stoll, as well as of the absence in our work of data on a differentiated identification of carotinoids, the results obtained by us should be considered as preliminary ones, furnishing some idea of the order of the values.

A determination of the content of carotinoids in air-dry specimens of silts has been made in samples of the bottom deposits of the following lakes: Balkhash, Balpash, Sultan-Kul, Etes, Sviatoie (peat). Analogous determinations in freshly taken (wet) specimens have been made in silts of the lakes: Beloie, Piavochnoie, Kolomenskoie, Glubokoie, Biserovo.

I. Air-Dry Specimens of Silts. Two specimens from lake Balkhash, taken on the shore of the Alakul bay, represent typical specimens of balkhashite, strongly oxidized. In neither specimen have carotinoids been discovered.

Lake Balpash. It is located 30 km from the health-resort of Borovoye in North Kazakhstan; its area is about 1 km^2; its greatest depth, 25 cm. It is a salt lake; the upper layer of silt is of red colour; in the plankton there are lots of *Artemia salina*. In the specimen taken from the surface of the silt deposits. 1.5 γ of xanthophyll in 1 g of air-dry silt have been found.

Lake Sultan-Kul. It is located on the right bank of the Ural river, 30 km south of Magnitogorsk; the depth is 1 m. The silt is of a great thickness, over 3 m. A typical calcareous gittia (42% CaO); in the lake there is a large amount of *Chara* sp. The water is fresh. Xanthophyll has been found in an amount of 32.7 γ per 1 g of air-dry silt.

Lake Etes. It is located in the foot-hills of the Kyrty range in south Urals, 40 km from Magnitogorsk. Its area is about 20 hectares; depth 1 m. The thickness of the silt is over 1 m. Calcareous gittia (25% CaO). Traces of carotin have been found.

Lake Sviatoie. It belongs to the group of the lakes at Kosino near Moscow. It has been subjected to a detailed study by a number of authors (see «Transactions of the Kosino Biological Station» and «Transactions of the Limnological Station at Kosino»). Specimens of dry peat from the shore of the lake have been studied. No carotinoids have been found.

II. Freshly Taken (Wet) Specimens of Silts. Lake Piavochnoie. It belongs to the group of the Zaluchino lakes located in the region of the Sapropelic Station of the USSR Academy of Sciences. Studied by Sturm and Simakova (1920), Petrov (1926), Orlova (1923), and others. A typical peat window surrounded by dense moss. Depth up to 0.8 m. Only traces of carotin and xanthopyll have been found.

Lake Kolomenskoie. Also belongs to the group of the Zaluchino lakes; area 5.2 km^2; depth up to 5 m. The amount of carotin was found to be 117 γ, and that of xanthophyll 78 γ to 1 g of absolutely dry silt.

Lake Glubokoie of the Moscow region in the area of the Hydrobiological station of this lake [a description see in the «Transactions» of the

station, particularly on morphometry see Muraveisky (1931)]. Area 60 hectares, greatest depth 32 m. No carotinoids have been discovered.

Lake Biserovo. Located near station Kupavna of the railway main Moscow—Gorki. Area about 100 hectares, greatest depth 4.5 m. It has been studied by Skadovsky, Savich, Briukhatova (1928), Soloviev (1932), and others. The bottom is covered by a thick layer of silt. Every year it is blooming with *Myxophyceae* and *Protococcaceae*. With respect to the content of carotinoids in the silts of the lake the following data have been obtained (per 1 g of absolutely dry silt):

Date	Carotin	Xanthophyll
May 19, 1936	1.67 mg	2.62 mg
June 8, 1936	0.99 »	1.48 »
June 30, 1936	0.44 »	0.85 »
June 30, 1936	0.45 »	1.11 »
June 30, 1936	0.33 »	0.60 »

On June 30 specimens were taken at different points of the lake at a depth of 4 m. It was observed that the amount of carotinoids in the silts of lake Biserovo, as determined in analysis, largely depends on the method of taking the sample; in samples taken with the aid of Ekman-Berge's dredger a larger content of carotinoids has been found than in those taken out with Petersen's dredger, which is heavier and sinks deeper into the silt. This suggests a larger content of carotinoids in the most superficial layer of the silt.

In December 1936 three samples of silt were taken (under the ice) and handed over to be analyzed by Doctor Lederer (The All-Union Vitamine Institute). According to the colorimetric determination, sample № 1 contained 0.2 mg of carotin. From the carotin fraction of silt № 1, which had very indistinct absorption bands, a certain amount of crystals (about 0.2 mg), probably of carotin, was obtained.

In February 1937 one sample more of silt was taken and sent over also to Doctor Lederer. In 90 g of dry residue he obtained 1 mg of carotin in a crystalline form; probably, it was a mixture of $\alpha + \beta$ carotin. In the group of hydrocarbons there are two pigments more, which are probably monoketones and resemble mixoxanthine and echinenone. From the fraction of xanthophylls, crystals of two pigments have been obtained (0.5 mg). A precise identification was not found possible. In addition to these two xanthophylls there are about 5 or 6 other pigments in this fraction. All of them have about the same absorption spectrum (477.422 μμ in petroleum ether) which corresponds to the spectrum of lutein. The amount of carotin is 7 mg, that of monoketones in the extract is approximately 1 mg, that of xanthophylls is 15 mg.

A complete extraction of pigments is very difficult and hardly feasible. According to Lederer, the production of carotin from this silt cannot be of any economic importance.

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GENETICS

STUDIES ON POLYPLOID PLANTS. PARALLEL VARIATION

By G. D. PRATASSENJA

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Experimental production of polyploid plants and their close study may throw light upon the problems of plants phylogeny. It is demonstrated by observations made in recent years. As an example may serve the data by Gregor and Sansome ([10]) on experimental production of a hexaploid *Ph eum*, closely related to the one existing in nature; those of A. Müntzing ([12—13]) who obtained a species of *Galeopsis* existing in nature as a result of crossing of two other species; those of V. Rybin ([6]) for *Prunus domestica* and *Prunus insititia*; of M. Toumanian ([7]) on production of tetraploid *Triticum monococcum* which received the name *Triticum jerevani*, a species which is closely related to *Triticum dicoccoides*. Unfortunately many studies devoted to experimental production of polyploid plants lack detailed descriptions of alterations in all parts of the new polyploid plants. Comparing the changes occurring in different parts of the plant together with doubling of the chromosome number under experimental conditions in different species both closely related and remote, parallel variation of homologous characters is observed (size of chloroplasts, of cells, of flowers, of seeds, and so on). The problem of parallel variation attracted great attention of Darwin ([2]) and many other authors. Detailed treatise of the problem of parallel variation is found in the work by N. I. Vavilov ([1]) in a general form of the «Law of homologous series in hereditary variation».

In our communication attention is concentrated upon parallel variation of homologous characters in plants belonging to different species when this variation is due to tetraploidy produced under strictly controlled experimental conditions.

We have observed in the first place like many other authors that in all autotetraploid plants experimentally produced the flower size is appreciably increased. An other no less important change observed in experimentally produced tetraploid plants but for some reason given inadequate attention by other writers is variation in sizes of seeds and chromosomes. Observations on variation connected with doubling of the chromosome number were conducted in autotetraploids of *Pelargonium roseum*, *Solanum lycopersicum*, *Nicotiana glauca*, as well as in amphidiploids of *Nicotiana rustica* × *N. tabacum* and *N. tabacum* × *N. glauca*. All the tetraploid plants enumerated above except for the amphidiploid *N. tabacum* × *N. glauca* have been obtained through doubling of the chromosome number in somatic cells of the plants following regeneration ([4], [5]). The amphidiploid *N. tabacum* × *N. glauca* has been obtained by Ternovsky by means of hybridization.

Comparing the seeds of autotetraploids with those of the original forms it is easy to see that seeds of autotetraploids exceed in size those of the original diploid plants (Fig. 1). In order to make sure of the reality of the size differences seeds of all polyploids and the original forms available have been weighed. The results are presented in the table. Previous to making comparative determinations of weight, seeds of all the *Nicotiana* were immersed in water. When immersed, seeds which were externally quite indistinguishable segregated into two groups: seeds belonging to the one group sunk to the bottom, while others remained at the surface. Floating seeds consisted of the seed-coat only being totally devoid of contents. Empty seeds externally indistinguishable from the filled ones were abundant in autotetraploids of *Nicotiana glauca* and in amphidiploids of *Nicotiana rustica*×*N. tabacum*, and *N. tabacum*×*N. glauca*, while in the original species *N. tabacum*, *N. rustica* and *N. glauca* nearly all the seeds were filled. Out of a thousand seeds of diploid *Nicotiana glauca* 988 seeds sunk, and 12 only have remained at the surface. The seeds which sunk were removed from water and dried under identical conditions at room temperature during 5 days whereupon they were weighed. One thousand seeds of *Nicotiana glauca* autotetraploid weighs 0.0349 g prior to separation of empty seeds, while after separation the weight rises to 0.0634 g. One thousand seeds of amphidiploid *Nicotiana tabacum*× ×*N. glauca* weighs before empty seeds are separated 0.0379 g, while one thousand of filled seeds weighs 0.0944 g.

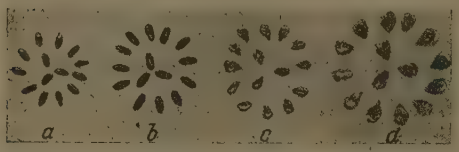

Fig. 1. Seeds: *a* — diploid of geranium, *b* — autotetraploid of geranium, *c* — diploid of tomato, *d* — autotetraploid of tomato (natural size).

	Pelargonium roseum		*Solanum lycopersicum*		*Nicotiana glauca*		Amphidiploid *N. rustica* × *N. tabacum*	Amphidiploid *N. tabacum* × *N. glauca*	*Nicotiana rustica*	*Nicotiana tabacum*
	Diploid	Tetraploid	Diploid	Tetraploid	Diploid	Tetraploid				
Number of seeds weighed	14	2000	137	376	1000	1000	1000	1000	600	1000
Weight of seeds in g	0.04	10.00	0.4374	1.794	0.0494	0.0634	0.1828	0.0944	0.2280	0.082
Calculated weight of 1000 seeds	2.857	5.00	3.193	4.558	0.0494	0.0634	0.1828	0.0944	0.380	0.082

As may be seen from the table, the weight of autotetraploids is greater than that of the original diploid forms. As to amphidiploids, the weight of their seeds is, in spite of the increase of the chromosome number, not always greater than that of the species possessing larger seeds from the two entering the cross, resulting in the production of the amphidiploid. This is evidenced by the results of weighing seeds of the amphidiploid *Nicotiana rustica*× ×*N. tabacum* when they are compared with the seeds of the original species, *Nicotiana rustica*. In the amphidiploid *Nicotiana rustica*×*N. tabacum* full diploid sets of the two parental species, *Nicotiana rustica* and *N. tabacum*, are present, while the weight of seeds of *Nicotiana rustica* exceeds that of the seeds of the amphidiploid *N. rustica*×*N. tabacum*. On the other hand, the

weight of the seeds of the amphidiploid *N. tabacum*×*N. glauca* exceeds the weight of the seeds of each of the two species involved in the cross when taken separately. These data enable one to make the following conclusions: 1) parallel to the chromosome doubling in all our species an increase in the size of the seeds takes place in case of autopolyploidy. The increase of the size of seeds in contrast to the slight increase of the vegetative mass in tetraploids as compared with diploids points out Randolph ([14]) for Indian corn. One thousand kernels of diploid Indian corn weighs 221.6 g, while in the tetraploid the corresponding value attains 330.2 g. 2) The change in size of seeds in amphidiploids as compared with the original species is not always proportional to the change of the amount of chromatine in the nucleus. Parallel to the chromosome doubling in hybrids *Nicotiana rustica*×*N. tabacum* an increase in the size of the seeds takes probably place. Unfortunately we are unable to compare the seeds of the amphidiploid *Nicotiana rustica*× ×*N. tabacum* with those of the amphihaploid produced by the same cross owing to its full sterility in our experiments. 3) The size of the seeds of the amphidiploids when seeds of the original species entering the cross greatly differ in size, may be less than that of the seeds of the species possessing larger seeds from the two entering the cross.

Randolph is inclined to explain the increase of the size of seeds in tetraploid Indian corn as compared with the seeds of the diploid by a better nutrition of the seeds of the tetraploid due to incomplete filling of the ear of Indian corn. This opinion was not verified by the investigation of the relation between the weight of the seeds and their quantity per capsule in the amphidiploid *Nicotiana rustica*×*N. tabacum*, as well as per inflorescense in diploid and tetraploid *Pelargonium roseum*.

Anyhow the fact deserves notice that, parallel to chromosome doubling, there takes place an increase in the size of seeds in *Pelargonium roseum*, *Solanum lycopersicum*, *Nicotiana glauca*, which belong to different families, and, when Randolph's data on corn be taken into consideration, to different classes (*Mono-* and *Dicotyledoneae*), which was also noted by N. I. Vavilov ([1]).

An attempt to compare the chromosome number and their volume with the size and weight of the seeds has been made by Ishii([11]) in *Dianthus*. Those data gave no clear correlation. The correlation observed in plants possessing low chromosome numbers is disturbed when compared with plants having many chromosomes.

In studying external changes of plants as connected with changes in caryotype in *Ononis reclinata*, Kartaschova ([9]), while comparing *Lotus angustissima* (12 chromosomes) with *Lotus hispidum* (24 chromosomes), did not find that the tetraploid species and varieties of the plants named above possess larger sizes of vegetative and generative organs as compared with diploids. These cases may be evidently explained from the data obtained in case of the amphidiploid *Nicotiana rustica* × *N. tabacum*, where as already noted seeds are smaller than in the one of the original species, in spite of the higher chromosome number. However, some authors in *Trigonella*, as well as Finn ([8]) in *Monogyna* and Ishii ([11]) in *Dianthus*, are of the opinion that with the increase in volume of the chromosomes (Finn, Ishii) and with doubling of the chromosome number (Ishii) the sizes of various organs are increased.

In all species investigated by us a parallel increase of the size of seeds is firmly established for all experimentally produced autopolyploids connected with the doubling of the chromosome number, while in amphidiploids, as compared with original species, in spite of the increase of the chromosome number, a decrease of the size of seeds sometimes occurs. Whether the cause of the decrease of the size of seeds is accumulation of chromatine beyond

limit, after which a decrease takes place, or an intermediate size between the sizes of the seeds of the original species is produced, is difficult to decide upon. More likely the second alternative is true.

Cytological investigation of the autotetraploid *N. glauca* brings to the light the fact that chromosomes are greatly diminished as compared with its diploid form. An identical decrease of the chromosome size following doubling of their number was observed earlier in *Pellargonium* [see fig. on page 9 (4)]. Owing to the fact that no data on temperature at the beginning of fixation of the material for cytological investigation were taken into account, it could not be decided upon changes in chromosome size connected with doubling of their number.

Plants of diploid and autotetraploid *N. glauca* were recently cultivated under identical conditions. Fixation of material for cytological analysis was done under application of one and the same fixative and simultaneously. Further examination was conducted with identical chemicals and under application of one and the same mounting of the microscope and drawing apparatus. As may be seen from Fig. 2, the chromosome size in diploid *N. glauca* exceeds that of the chromosomes in autotetraploid *N. glauca*.

The latter circumstance supports on a material of certain origin the suggestion of Levitsky and Kusmina(3) as to the decrease of chromosome size parallel to their number increase in the genus *Festuca* and contradicts the data by Finn(8) for the section *Monogynella* and *Callianche*, where, according to his opinion, the process developes in the direction of increase of the chromosome mass.

If in discussing data by Levitsky and Kusmina one takes into consideration the indications(5) on depressed condition of autotetraploid *N. glauca*, as compared with the original form, it should be, neverthelles, kept in mind that the majority of progeny of this autotetraploid exceeds in size the diploid *N. glauca*. These changes in the chromosome size justify the conclusion as it was earlier suggested [Kartaschova(9)] that we not only observe the phenomenon of doubling of the chromosome number, but also more profound changes of the caryotype which are expressed in alteration of chromosome sizes and shapes and lead to a whole series of morphological, physiolpgocal, chemical and other changes. Morphological examination of the progeny of experimentally produced polyploids revealed a great diversity of different characters. The whole diversity observed in the progeny of the amphidiploid *N. rustica* × *N. tabacum* does not fit into any of the schemes known to us. We are inclined to ascribe this phenomenon rather to mutability of plant cells, both in soma and in generative organs, than to segregation of the polyploid obtained.

Bud mutations were often observed. The shoots from the mutant buds possessed flowers with entirely different colour of the corolla, shape and coloration of the calyx, etc., than the flowers of the basic plant. A detailed account of this phenomenon will be given elsewhere We shall dwell here upon the character of the arrangement of side veins of the lamella. It is interesting in that respect that under experimental conditions a parallel change of homologous characters takes place in plants belonging to two entirely different species as a result of doubling of the chromosome number, which supports assumptions worked out by N. I. Vavilov(1).

One of these species is the amphidiploid *N. rustica* × *N. tabacum* ($n=48$)(4), and the other—the autotetraploid *N. glauca* ($n=24$)(5). In the progeny of the amphidiploid *N. rustica* × *N. tabacum*, as was noted above, a great diversity of forms was obtained. Aside from other forms in the third generation, forms of plants were discovered in which side veins of the leaves make with the central vein a very acute angle (Fig. 3, *c*), and the petiole very often

enters the lamella not with a single vein like in the original forms, but with a bunch of parallel veins which, after the entrance of the petiole into the lamella, «fanlike» diverge towards the edges of the lamella. However, the main vein of the lamella is thicker than the side veins.

The same appearance of arrangement of side veins of the lamella may be seen in the autotetraploid *N. glauca* (Fig. 3, *d*, leaf of autotetraploid; 3, *b*, that of a diploid), with the only difference that no segregation in this character was observed in the progeny of the autotetraploid.

In the cross of the amphidiploid *N. rustica* × *N. tabacum* with the diploid *N. glauca* the arrangement of veins of the lamella of diploid *N. glauca*

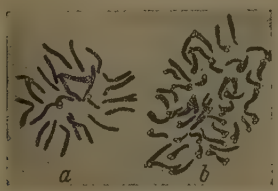

Fig. 2. Somatic plates: *a*—diploid *N. glauca* and *b* — autotetraploid *N. glauca*.

Fig. 3. Leaves: *a*—*N. tabacum*, *b*—*N. glauca*, *c*—amphidiploid *N. rustica* × *N. tabacum*, *d*—autotetraploid *N. glauca*.

is completely dominant. In crosses with the autotetraploid *N. glauca* in F_1 are found plants with «fanlike» arranged veins of the lamella as well as those analogous to plants obtained in crosses with the diploid. These data enable us to speak of the recessiveness of the character of the «fanlike» arrangement of the side veins of the lamella in polyploid plants obtained.

Conclusions

1. With the doubling of chromosome number in autopolyploidy the size of the seeds suffered parallel increase in *Pelargonium roseum*, *Solanum lycopersicum* and *N. glauca*, and in the amphidiploid *N. tabacum* × *N. glauca*.

2. In the amphidiploid *N. rustica* × *N. tabacum*, in spite of the increased chromosome number as compared with *N. rustica*, seeds are smaller than in *N. rustica* (the size is intermediate between *N. rustica* and *N. tabacum*).

3. With the doubling of the chromosome number in *N. glauca* the chromosome size was diminished.

4. With the chromosome doubling in *N. glauca* and in the hybrid *N. rustica* × *N. tabacum* a parallel appearance of a recessive character of the «fanlike» arrangement of side veins of the lamella took place in the two polyploids.

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GENETICS

PRODUCTION OF POLYPLOID PLANTS. A TRIPLOID IN *PRUNUS PERSICA*

By G. D. PRATASSENJA and E. M. TRUBITZINA

(Communicated by N. I. Vavilov, Member of the Academy, 1. IV. 1938)

In studying flower-buds of peaches cytologically (varieties «Téton de Vénus», «Belle de Vitry», «Bon Ouvrier», «Karl Inguf» and others) Kobel (5) established the fact that some of the pollen mother-cells divide, like somatic cells, just once, which results in formation of two-celled «tetrads» possessing diploid nuclei and producing further on giant pollen grains. Karpechenko (4) presented an analysis of diad formation and explained the origin of their diploid nuclei. The two authors note the possibility of obtaining from such gametes of plants possessing increased chromosome number. At the present time a whole series of works is known where formation of gametes containing somatic chromosome numbers and production of polyploid plants were attained by means of experimental influences upon the flower-buds. It was established (7) that polyploid plants are more resistant to severe conditions. The variety «Krassa Severa» produced by Michurin (3, 6) had probably originated under participation of an unreduced gamete of *Prunus avium* in crossing with *Prunus cerasum*.

In view of the above data it was decided to obtain polyploid plants of *Prunus persica* for the production of new forms which might prove to be more cold-resistant and, therefore, suitable for cultivation in northern regions.

The method of the present work was based upon the following: a) isolating of varieties which possess the maximum quantity of diploid pollen; b) selection of diploid pollen; c) artificial pollination by diploid pollen in the hope that union of two diploid gametes or of a diploid gamete with a haploid may produce, correspondingly, tetraploid or triploid plants.

Flower-buds were investigated at the tetrad stage.

Altogether hundred varieties of peaches were studied. In all the varieties studied tetrads were met; the percentage of the latter varied, however, from 0.1 to 18.9 according to the variety. Varieties possessing from 9 to 18.9% of diads were employed for the work («Fitzgerald»—18.9%, «Foster»—16, «Téton de Vénus»—16, «Königin Olga»—15, «Khidistavsky Dark-Red»—10.2, «Lord Palmerston»—10, «Leopold I»—10.1, «La France»—10, «Gorny Orel»—9.7 and «Salgirsky»—9%).

The analysis of pollen at the time of flowering of the plants has shown that pollen was unequal in size in the varieties employed, in agreement with the data of Kobel (5). Pollen could be easily divided into two groups,

containing small and large grains. One worker was able by means of a binocular and preparation needles to select a hundred of pollen grains in one hour.

Artificial culture of the large pollen grains has shown (the medium contained 10% of sugar) that the majority of them formed one pollen tube which was thicker as compared with those produced by normal pollen grains. Cases occurred when, as noted by Kobel ([5]), some of the large pollen grains developed two pollen tubes; more often it started as one single tube and after having attained a certain length began branching to form two branches.

Pollen was selected one day prior to pollination. Binocular microscopes with low stands were used by the worker for selection of pollen. Under the one binocular pollen (which was dried under room conditions) was examined, and the large grains were transferred by means of a needle to a glass placed under the other binocular; they were arranged in rows which was of advantage for the sake of control of the purity of the selected pollen grains. Before going to the orchard for the pollination of flowers which were previously emasculated, the selected pollen was gathered on glass needles made of glass cotton. The glass filaments were glued to thin enthomological needles by means of Canada balsam; the needles were previously pushed through corks used for stopping of excursion test tubes. Ten pollen grains were gathered on each of the glass needles which were prepared in the way described above; all the ten formed one group. Thus gathered the pollen grains were well seen by the naked eye; this made pollination easier. The glass needle bearing pollen was then placed in a test tube, and the stopper pushed in. Pollen could be thus easily transported from the laboratory to the garden; special boxes supplied with compartments for the tubes were used. Each stigma was pollinated with 10—30 pollen grains, that is with the quantity contained on one or three needles, depending of the germination of the pollen. The germination of the pollen was controlled before pollination on the 10% sugar solution.

In 1936 with diploid pollen were pollinated 985 flowers as a result of which 4 fruits were obtained. During the winter stratification in 1936/37 seeds rotted and gave no sprouts. In 1937 were pollinated with large pollen 900 flowers of peaches.

Eighteen fruits developed from this pollination; 16 of them were collected. Planting of the seeds was done immediately following the gathering of fruits. Four out of the 16 fruits were lacking seeds (parthenocarpy), 7 contained underdeveloped embryos, and only 5 fruits contained well formed embryos.

The stones were cautiously broken before planting, and the seeds taken out. The seed-coat and the perisperm were removed from the seeds following the indication of F. Flemion ([1], [2]) and R. Veh ([8]). Such a method of planting has the following advantages over planting of stratified seeds: on the one hand, seedlings are obtained a year earlier; on the other, seeds are given a possibility of maturing completely. Polyploid seeds which grow on diploid plants, as well as those of some of the early varieties of peaches (for instance, «Rivers' Early») are physiologically immature, while the fruit is marketable. Such immature seeds when taken off the tree lose germinating power after the slightest drying out; we made sure of this in studying seeds of the variety «Rivers' Early». It was sufficient to leave seeds in juicy pericarps upon gathering of fruits to make the seeds dry out and lose their germinating power. Not wholly mature seeds probably need, aside from water, free metabolism (respiration) which is hindered or almost stopped in the gathered fruit or in a seed taken out. Embryos freed from the seed-coat and perisperm when placed on a wet filter-paper in Petri dishes at a temperature of 18—20° over the freezing point germinated on the 4th or

5th day following planting. Seeds of the variety «Rivers' Early» and those obtained from pollination with the diploid pollen were thus germinated by us. From the pollination with diploid pollen 5 embryos germinated. Rootlets were fixed for cytological investigation from 4 out of these 5 seedlings (one died soon after germination).

Cytological investigation of the 4 rootlets from the plants fixed has shown that two plants had 16 chromosomes each, and the other two—24 chromosomes.

It was thus established that by means of artificial pollination with selected coarse pollen it is possible to obtain in *Prunus persica* triploid plants.

Triploid plants were obtained from the crossing of the varieties «La France»×«Fitzgerald» and «Kushnarev»×«Téton de Vénus».

The work on the analysis of varieties, on the selection of pollen and on the pollination was done by S. I. Elmanov and E. M. Trubitzina.

Conclusions

1. Coarse pollen was selected in several varieties of peaches and artificial pollination was done with it.

2. Upon artificial pollination with the selected pollen triploid plants were obtained in varieties of *Prunus persica*.

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GENETICS

SHORTTAILEDNESS (BRACHYURY) IN GEORGIAN DOGS AND ITS INHERITANCE

By M. RCHEULISHVILI

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Short-tailed and tailless forms are known in a number of species of domesticated mammals—rats, mice, guinea pigs, horses, cattle, sheep, cats and dogs, and may serve as a good example of parallel mutational variability.

However, the mutational origin of shorttailedness (and of taillessness) has been recorded with exactitude only on mice [Lang (9), Dobrovolskaia-Zavadskaia (3)], whereas in other species such an origin of the character is assumed as the most probable.

Lang (9, 10) had studied the hereditary nature of shorttailedness (and taillessness) in mice and came to the conclusion that the shorttailedness in this particular case is called forth by a single dominant gene, which occurs only in the heterozygous condition. This can be explained by the lethal effect of the gene of shorttailedness in the homozygous condition [Filipchenko (6)]. The considerable variation of abbreviated tails the author explains by the effect of additional polymeric factors (modifiers).

Lang has also ascertained that short-tailed individuals, as compared to animals with normal tails, are less fertile and that the tailless forms are altogether sterile.

Another mutational short-tailed type of a mouse, also dominant and lethal when homozygous, was obtained by Dobrovolskaia-Zavadskaia (3). By crossing members of this short-tailed strain to wild mice, there were obtained 3 new substrains, each characterized by complete absence of the bony tail (anury); the latter in turn by inbreeding within their own line would always produce only tailless animals [Dobrovolskaia-Zavadskaia and Kobozieff (4, 5)].

P. Chesley and L. Dunn (2) have shown that these true-breeding tailless forms bear two genes T and t°, evidently allelomorph to each other, each of them producing a lethal effect in the homozygous condition; consequently, tailless forms when bred true inter se contain in this case a balanced lethal system (Tt°—tailless forms, TT—abnormal embryos, dying on the 11th day after fertilization, $t^{\circ}t^{\circ}$—abnormal embryos, dying somewhat earlier (on the 7th day), T_{+}—short-tailed, $t^{\circ}{}_{+}$—normals). So far this has been the unique case of balanced lethals in mammals proved by experiment.

In 1909 W. Bateson recognized that taillessness in cats was a character incompletely dominant over the normal condition. Castle (1916) maintained that in such a case the homozygotes are tailless and the heterozygotes have abbreviated tails (6).

Upon summing up the facts, Bamber ([1]) finds the available material on cats to be insufficient and obsolete, and therefore more investigations are necessary to break new ground in regard to the inheritance of shorttailedness and taillessness in this animal.

According to the data reported by Vilmorin ([11]) and Klodnitzky and Spett ([8]), taillessness in dogs is contingent upon one dominant gene with a lethal effect (quoted from Filipchenko, 1928).

Considerable variation of the length of the tail is observed among sheep, and this character, along with the nature of formation of fat in the tail, is used as a basis for the classification of breeds of domestic sheep of to-day. Shorttailedness and taillessness in sheep is not connected with the lethal effect and does not decrease viability and fertility of bearers of these characters.

Hereditary shorttailedness and taillessness in other species of mammals have not yet been studied.

Considering the fact that both in mice and in dogs genes of shorttailedness and taillessness simultaneously cause a decrease of viability and fertility, it seems important to acquire new evidence on the heredity of these peculiarities in various species.

This work is devoted to the study of inheritance of shorttailedness in Georgian dogs, on the basis of a small experiment carried out by us and data collected during the investigation of dog breeding in Georgia.

While observing Georgian sheep-dogs (a strain of the Caucasian sheep-dog) in Eastern Georgia in 1936, we have not come across any tailless forms whatsoever, nor have we heard from the people anything about the existence of such, but out of a total of 317 dogs studied eight (i. e. 2.52%) had abbreviated tails.

M a t e r i a l. In 1932 I received from the district Sygnakh one short-tailed male dog (called «Alasha») of the Caucasian (Georgian) sheep-dog breed.

Alasha standing 65.5 cm at the withers, had a tail 22 cm long on the upper curve; thus, the length of the tail equalled 33.58% of the height up to the withers. The height up to the withers of males of the Georgian collie breed equals on the average 62.68 ± 0.27 cm ($\overline{n}=151$), and the length of the tail is 42.18 ± 0.15 cm ($\overline{n}=103$), that is, the length of the tail is 67.29% of the height up to the withers. Alasha's tail was 52.15% against the normal tail of Georgian collies (♂♂), i. e. about half of the normal size of a tail.

Among the short-tailed progeny of Alasha, a certain variation in the length of the tail was noticeable, but in general we failed to see among the Georgian collies such a great variation of this character as described by Lang ([9], [10]) in mice, by Vilmorin ([11]), Klodnitzky and Spett ([8]) in dogs.

Among the dogs measured during the expedition, it was discovered that the tail of the most short-tailed dog constituted 35% of the length of a normal tail, whereas the tail of the most long-tailed of dogs with inborn shorttailedness was 54% of the length of a normal tail. The shortest tail constitutes 23% and the longest 35.85% of the height up to the withers, whereas the similar index in normals varies from 45% to 82.87%.

From the shepherds of Kizikia I gathered the following information on Alasha's pedigree.

About 1917—1919 a bitch of a Kizikian shepherd (of the Sygnakh district) brought forth 9 puppies, of which 8 were normal-tailed and 1 was short-tailed which the shepherds did not fail to notice. And that was the puppy which grew to be Alacha's grandsire. As the shepherds from the vicinity stated, never before the birth of Alasha's progenitor had they known of a case of a short-tailed form having been produced among their dogs.

Taking into account the dominant and the monohybrid nature of hereditary shorttailedness (see below) and the rare occurrence of this character among the Georgian sheep-dogs, the arising of a single puppy in a large litter of 9 offspring, makes its mutant descension from Alasha's ancestor appear quite probable.

Crossbreeding. In the same year (1932) Alasha was crossed with normal-tailed bitches: with Karasa, with Fokousa* and then with Brolya and once more with Karasa.

These crosses yielded 24 offspring, which can be divided as follows, according to the length of their tails.

Name of mother (normal-tailed)	Offspring						
	Number of young in a litter	Short-tailed			Normal-tailed		
		♀♀	♂♂	Total	♀♀	♂♂	Total
Karasa	5	1	2	3	2	—	2
»	8	2	1	3	2	3	5
Fokousa	2	—	1	1	1	—	1
Brolya	9	2	4	6	—	3	3
Total { Actual	24	5	8	13	5	6	11
Total { Expected (1 : 1) : 24		6	6	12	6	6	12

It is evident therefrom that the short-tailed Alasha is a heterozygote and that the short-tailed character is dominant over the normal; this fully agrees with the data of Klodnitzky and Spett (4).

The found ratio 1 : 1 proves the monofactorial nature of the Brachy inheritance.

In 1936 in a country place called Girevy, in the mountains of Tushetya, at a villager's by the name of Yogouridze, I saw one short-tailed bitch with its 9 offspring. Of the latter 4 (2 ♀ and 2 ♂) were short-tailed and 5 (3 ♀ and 2 ♂) normal-tailed. Yogouridze assured me that he had never kept any short-tailed females, except this one bitch, and that the male parent of the puppies was a normal-tailed dog.

Thus, the distribution of the young in the litter of Yogouridze's bitch is in perfect agreement with our data, this too being the case of a cross between a dominant heterozygote and a recessive homozygous individual.

Summarizing all the described crosses, we may establish that of the total of 33 puppies 17 were short-tailed and 16 normal-tailed.

The ideal ratio should have been 16.5 : 16.5 with a margin for an average error of ± 2.87. The difference between the ideal and the actual figures—0.5, as compared to the allowable error (± 2.87) is immaterial. The obtained ratio 1 : 1 testifies to the monofactorial nature of shorttailedness and the heterozygosity of short-tailed individuals.

Our data are in good agreement with those communicated by the agronomist Kandelaki, in his report delivered at Tbilissi. Kandelaki crossed a short-tailed male dog of the Caucasian collies stock with a normal-tailed

* Both Karasa and Fokousa belonged to the experimental sheepfold and descended from normal-tailed dogs, and according to the shepherd's assurance, among the immediate ancestry of this stock a short-tailed dog has never been produced.

female hound, and the cross yielded a litter of 4 normal-tailed and 4 short-tailed young. He further mated one short-tailed male of the aforesaid offspring with a normal-tailed bitch of the Irish setter breed and that cross yielded 3 short-tailed and 3 normal-tailed puppies. Thus, Kandelaki obtained in both crosses an absolutely exact concurrence of expected and actual numbers.

The sex ratio in groups of normal-tailed and short-tailed animals, close to a normal one, speaks for the inheritance of this character regardless of sex.

We have no materials on inbreeding of short-tailed dogs inter se at our disposal, therefore the question remains unsolved whether the gene of short-tailedness in Georgian dogs is lethal when homozygous or not. All heterozygous individuals are fully viable, and in this respect they are not inferior to normal-tailed ones. The short-tailed dog Alasha and Yogouridze's bitch, as well as the two dogs used by Kandelaki for the purpose of interbreeding, turned out to be also fully fertile.

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PLANT PHYSIOLOGY

ENDEMIC CONDITIONS OF GROWTH OF *GOSSYPIUM KIRKII* MART.

By V. A. NOVIKOV

(Communicated by A. A. Richter, Member of the Academy, 4. IV. 1938)

Gossypium kirkii is a wild cotton plant and as such deserves our attention, since a study of wild species will contribute to a rightly established classification. And if some of the investigators eliminate them from the genus *Gossypium*, nevertheless all are unanimous in recognizing wild cotton as closely related to the cultivated ones. But there are more good reasons to take interest in them. As Mauer (3) has pointed out, the wild species possess many characters and properties highly valuable for the practice; such as fruitfullness, endurance under severe conditions of life (drought and frost resistance), strong resistance and immunity with regard to the chief diseases and insects attacking cotton. Sometimes they are distinguished by a thin tough and smooth fibre, etc. All that makes them a valuable original material for interspecific crosses. The possibility of crossing wild cotton species with cultivated ones may now be considered firmly established owing to the experiments carried out by the Central Breeding Station of the All-Union Cotton Research Institute and by the Azerbaidjanian and Tadjic Cotton Research Institutes. Such crossing is widely applied in the breeding entreprises of the country being one of the starting points for practical breeding and creation of new valuable cultivated varieties of cotton.

Experiments with cultivating *G. kirkii* made at the Central Breeding Station both in field and hot-house conditions, ended in failure.

The station of this plant is scarcely known (5). Kearney (1) reports that the conditions under which it was found to grow were of undoubtedly endemic nature. But in what were they endemic reported neither he nor other investigators. Konstantinov (2) supposes that the endemic conditions of *G. kirkii* consist in its need of much water. He has not been successful, however, in growing *G. kirkii* under conditions of high moisture.

To throw light upon the condition favouring the growth of *G. kirkii* I undertook the following experiment. On the 26 of May 1935 at the Central Breeding Station I set four plants at the stage of cotyledons into four vegetative vessels each containing 20 kg of absolutely dry soil. The soil to fill the vessels was fertilized according to the rate of 4 g $N + 6$ g P_2O_5 per vessel, i. e. per 20 kg of soil. The moisture of the soil throughout the experiment was 60% of ito full moisture capacity.

The plants took root but grew poorly. The leaves fell off before being fully developed. No more than one leaf remained on each plant. In July with setting in of dog-days the suffering of plants became worse. On July

15 one plant perished; on July 20, the second; on July 23, the third. The fourth plant which survived looked very poor.

By that time became apparent the results of another experiment which was carried on simultaneously and had for object to discover the effect of light intensity on the development of cotton. It showed that variety Navrotsky had grown better in weakened sunlight than in full light (5). This led to suppose that bright sunlight might be the factor which restrained the growth of *G. kirkii* under conditions prevailing at Tashkent. The survived plant was therefore on May 25 put under one layer of gauze. The intensity of light was thus reduced to 47% of full daylight.

As early as on the th rd day new leaves appeared on the plant transferred under gauze, the single old leaf continuing its life too. By the 20 of September about 20 leaves were recorded on the plant. The stem, however, grew very slowly and by that time was only 12 cm high. On September 20 the plant was transferred to the hot-house, where it remained until May 20,

Fig. 1.

1936. On this latter day it was again put under gauze, but two layers instead of one were stretched over it. The experiment carried out in 1935 [Novikov(4)] had shown that cotton develops better when the intensity of light is 20% of full intensity, which had been obtained by stretching two layers of gauze on the frame.

Put under gauze the plant quickly recovered. Foliage lost in hot-house was restored and the plant grew visibly larger. Its height measured every five days gave the following figures:

15.VI	25.VI	5.VII	15.VII	25.VII	5.VIII	15.VIII	25.VIII	25.IX
25.0	26.5	27.5	28.0	29.5	33.0	37.0	40.0	52.5

The growth in height was attended by an intensive formation of lateral branches which, the same as the main stem, gradually increased their length. By the end of September the plant looked like a full-formed shrub, rich in foliage (Fig. 1).

The usual effect of a weakened light is that the lengthening of the cells in a plant is going on more intensively than the thickening of their walls. That is the reason why plants gr wn in the shade have elongate, thin stems, easy to broke.

When growing the cotton *G. hirsutum* variety Navrotsky an increase of growth was observed with light intensities equal to 47% and 21% of

the full daylight. This increase, however, was not attended by thinning and feebleness of the plants which in spite of having grown in a shaded light were strong and developed best, when the light was reduced to 21% of the full daylight.

In 1936 alongside of the experiment on *G. kirkii* the cotton *G. hirsutum* variety № 8517 was grown in shaded light of various intensities. Its

Fig. 2. *Gossypium hirsutum* № 8517. Control, 47% and 21% light intensity (left).

behaviour was similar to that of variety Navrotsky in the experiment of 1935. The plants of that variety which were grown in light of different intensities are shown in Fig. 2 (photo taken on September 29). It may be seen from the photograph that the plants which were given less light grew more intensively. But by no means can we say that the intensified growth resul-

Fig. 3. On the left *G. hirsutum*, on the right *G. kirkii*.

ted in feebleness of the plant. On the contrary, the plants grew stronger in shaded light as it had been the case in the experiment with the variety Navrotsky. If we compare, however, the plants of *G. hirsutum* with those of *G. kirkii*, grown at the same light intensity—21% of the full daylight, we see that the shrub of *G. hirsutum* is looser and thinner that that of *G. kirkii* (Fig. 3).

The data on growth referred to above and the photographs 1 and 3 show that even decreasing the daylight 5 times we see *G. kirkii* to grow, yet very slowly. If we consider this evidence alongside of the fact that the plant suffers badly under full sunlight, we may conclude that the weak intensity of light is the chief endemic condition of growth required by *G. kirkii*.

In 1937 N. N. Konstantinov, of the Central Breeding Station, succeeded in growing *G. kirkii* which gave fruits and had 11 bolls completely matured. The plant was given a curtailed (10-hour) day of reduced intensity.

The chief endemic condition of growth of *G. kirkii* may now be considered established. It consists in lowered intensity of light (50—20% of the full daylight at Tashkent) promoting growth, and in shortened day necessary for the transition to reproductive development.

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PLANT PHYSIOLOGY

EFFECT OF HARDENING UPON STRUCTURE OF PLANT

By N. S. PETINOV and G. A. ZAK

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Plants when subject to drought acquire xeromorphous structure, which is made up of a series of morphological, anatomical and physiological peculiarities (Zalensky, 1901, 1904, 1920; Kaznov, 1918; Rippel, 1919; Lebedinzeva, 1926; Tumanov, 1926; Alexandrov and Ckhakaya, 1926; Maximov, 1926; Vasiliev, 1925, 1935; Kursanov, 1933, and others).

Other factors, however, besides that of soil dryness may account for the xeromorphous structure. Such are, for instance, excessive humidity of soil and air, low temperature and lack of mineral nutriment (Kokina, 1929), each of them inhibiting the growth of plants. Plants which suffer their action are distinguished by smaller cells, greater number of stomata per unit area, denser venation, etc. But it was not the structure alone which attracted the interest of investigators. For a long time attempts would be made to establish connection between structural elements and the yield of plants as well as intensity of their physiological processes under various conditions of water supply. Kolkunov, for instance (1905, 1907-a, 1907-b, 1910, 1913 and 1922) reported that the yield of plants grown in highly humid soil and atmosphere is proportional to the size of their cells. The rate of carbon assimilation and the amount of dry matter stored up by a plant in a humid climate corresponds to the length of its stomata (Kolkunov, 1910, 1913), while the crop yield varies in inverse proportion (Kolkunov, 1913). Later this author pointed out that for each complex of climatic conditions there exists an optimum of the size of stomata (Kolkunov, 1922). Further investigations carried out by Vasiliev (1925) have shown that the connection between the yield and length of stomata is a matter of fact in wheat, though it is differently expressed in various groups of this plant being dependent moreover on the conditions of their existence.

Of special interest are the works of Rywosch (1908), Alexandrov (1920, 1924) and Kursanov (1911) on the dependence between the structure of plants and their physiological processes. In this respect an important rôle is ascribed by Alexandrov (1924) to the vein-sheaths. As the latter seem to promote the outflow of synthesized assimilates and moreover present an assimilative system, the plant whose leaves possess a pronounced apparatus of sheathing should store up the dry mass more energetically. Indeed, a rapid outflow of assimilates from the parenchyma of the leaf should enhance the rate of its assimilating work considerably (Rywosch, 1908; Alexandrov, 1924; Kursanov, 1933, and others).

At the same time a well-developed sheathing of small veins in the leaf of a herbaceous plant is one of the causes increasing the effect of transpiration (Alexandrov, 1924). Later Kursanov and his associates pointed out that a certain dependence exists between the equipment of a leaf with conducting system and the photosynthetic energy. Thereon he founded his interpretation of the increased photosynthesis in xerophytous plants. According to his data, however, the conducting system in the first place owes its enlargement to the elements of phloem. The ratio of xylem to phloem decreases as the drought grows deeper. Thus, the energetic assimilation in xerophytous plants, aside from their being better equipped with water conveying system, is chiefly due to a better development of their phloem, i. e. easier outflow of assimilates.

Lately the works of Skazkin and Skazkina (1937) have established that drought differently affects the anatomical structure and yield of plants (oats) when the latter suffer its action during different periods of vegetation or belong to different biotypes. These authors observed a marked decrease in the size of cells, area of water conveying system and grain crop during the most critical periods of development: stalk-shooting—earing in highly drought resistant Anatolian oat, and tillering—stalk-shooting in Moscow oat, not resistant to drought. An exception form the first (sprouting—tillering) and the second (tillering—stalk-shooting) periods of drought in Anatolian oat; they reduced neither the area of water conveying system nor grain crop.

In our work* we deemed it important to trace the influence of a short drought exerted at different periods of development on the structure of a plant (summer wheat *Hordeiforme* 0189) in connection with physiological processes and grain yield.

M e t h o d. a) Study of openness of stomata. A celluloid film freed from gelatine coat was cut into pieces 5×10 mm on the average. The broad edge of each piece was bent up to grip it easily with forceps. One of the blades of the latter was soldered with an inflexible plate which fell on it perpendicularly and had the same dimensions as the piece of film, i. e. nearly 5×10 mm. The piece held with the forceps was smeared with acetone densely coloured with fuchsine alcohol solution by means of a pencil and quickly applied to the surface of the leaf. In this way a red line copy of the leaf surface with stomata was imprinted on the celluloid film which could be easily preserved in envelopes or beakers. The number of stomata (in the field of vision**) was averaged from 50 counts.

b) For the study of anatomical structure portions distant $^1/_3$ of leaf length from the base (ligule) were cut out and fixed. The stem portions were taken in the second internode from the top. Iuel's mixture was used for fixation (zink chloride, 2 g; icy acetic acid, 2 cm^3; 70% alcohol, 100 cm^3). Washing, dehydration and imbedding in paraffin followed in their usual way, after which sections 20 μ thick cut by means of microtome were stained with safranin and put into glycerin—gelatin.

R e s u l t s. When forming, the structure of plant, as every process of growth is, directly influenced by the environments and proves to be very sensitive (especially the leaf) to their influence throughout its development. Experiments showed that under the action of a short shallow drought (hardening) summer wheat acquires the usual characters of a xeromorphous structure; above all it enlarges its water conducting system. The fibro-vas-

* See previous articles I and II in C. R. Acad. Sci., XVIII, № 1 (1938), where the plan of the experiment has been described too. In I, p. 151, the term «thermal» stage has erroneously been applied instead of «yarovization» stage.

** Microscope Reichert, objective № 4, ocular IV.

cular bundles are closer in the stem (Fig. 1,2). Their number in a cross-section of a stem (second internode from the top) were the following:

1. Control—optimal 60% moisture throughout development . . . 31
2. Hardening at the outset of tillering 39
3. Hardening during tillering 43
4. Hardening during stalk-formation 42

There was also formed an increased number of stomata per unit area of the leaf, especially in plants which underwent hardening in their earliest period—outset of tillering (Table 1). In connection with different conditions of water supply and evaporation the number of stomata markedly changes along the height of the stem. The upper leaves, as shown by Zalensky (1911), develop

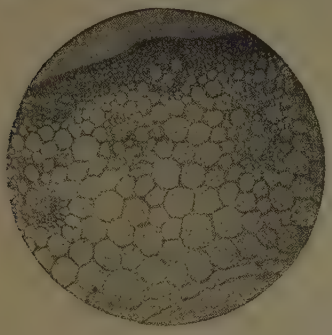


Fig. 1. Control—optimal 60% moisture throughout the experiment.

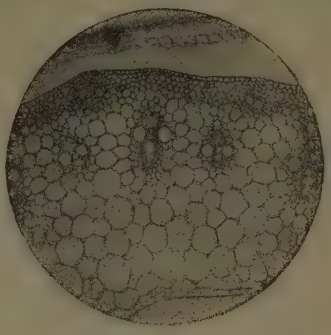


Fig. 2. Hardening at the outset of tillering.

Cross-Section of the Stem

under conditions of a curtailed water supply and stronger evaporation. On this account the number of stomata decreases from the upper leaves to the lower. We always found, too, a greater number of stomata on the upper side of the leaf than on its under side, which has already been pointed out by Jakushina and Vavilov (1912).

Table 1

Number of Stomata in the Field of Vision of the Microscope
(Summer Wheat *Hordeiforme* 0189) (Average from 50 counts)

Series of experiment	Whorl, from above	Side of leaf	
		Under	Upper
Control, optimal 60% moisture throughout the development	1	66	80
	2	44	53
	3	37	40
Hardening at the outset of tillering	1	70	83
	2	57	67
	3	51	54
Hardening during stalk-formation	1	66	82
	2	51	67
	3	45	47

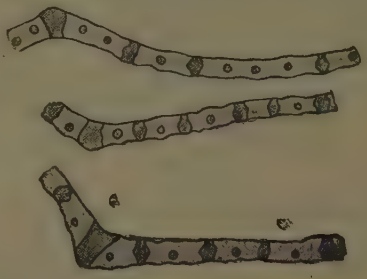
We have thoroughly examined the structure of fibro-vascular bundles in the leaves of our test plants. In wheat large vascular bundles alternate

with small ones. The large bundles consist of well formed xylem and phloem parts and of a strongly developed mechanical tissue which surrounds almost the whole bundle. The small bundles chiefly consist of phloem; the elements of xylem make usually up a little part in the cross-section of a small bundle. These bundles are, as a rule, surrounded with large cells of the sheathing (Alexandrov, 1924). The alteration of bundles varies from leaf to leaf (wheat *Hordeiforme* 0189), when the latter have been taken from different whorls or subjected to different influence of environment. In the lower leaves of controls (2d and 3rd whorls from the top) 2—3 (more often 3) small bundles lie between 2 larger bundles, while in the upper leaves of controls we in most cases found only one small bundle situated between two large ones; leaves which have undergone hardening show a changed alternation of bundles. In plants subjected to drought at the outset of tillering the lower leaves (2d and 3rd whorls) show 1, rarer 2 small bundles between 2 large ones, while the upper leaves do not differ from the controls. In plants hardened during stalk-formation there is observed a sharp transition from a sparce distribution of large bundles in the 3rd whorl to a dense one in the 2d and 1st whorls. Thus, hardening induced an increase in the ratio of large to small bundles in the wheat leaves of the 2d and 3rd whorls from the top (Fig. 3,4,5). The change

Fig. 3. Control—optimal 60% moisture throughout the experiment.

Fig. 4. Hardening at the outset of tillering.

Fig. 5. Hardening during stalk-formation.



Alternation of Large and Small Bundles (2d whorl from the apex)

of this ratio alters the course of physiological processes. As has been stated by Alexandrov (1924) and supported by our experiments, plants with leaves which have their small vascular bundles sheathed, possess a greater energy of photosynthesis and a greater productivity of transpiration. To gather and discharge the assimilation products, that is apparently the function fulfilled by the small bundles, consisting chiefly of phloem. The large bundles with strongly developed xylem have their main task in providing the plant with water. As a matter of course, it follows that an increase in the number of small bundles (owing to hardening of plants) and a respective decrease in the total number of cells reduce the productivity of transpiration* (Table 2).

We calculated the coefficient and productivity of transpiration for the total mass and grain crop separately. From Table 2 it may be seen that hardened plants show a high transpiration coefficient alongside of a low productivity of transpiration (reckoned to grain crop), i. e. they spend water in excess. With reference to total mass plants hardened during stalk-formation form an exception since their yield of vegetative mass differs but slightly from that of controls [see C. R. Acad. Sci. URSS, XVIII, 1 (1938)].

Attention should moreover be paid to the fact that not only the ratio between large and small vascular bundles but also their structure is subject

* Productivity of transpiration is the amount of dry matter (in grams) formed in the plant during vegetation per 1 kg of water absorbed (Maximov, 1926).

Table 2

Transpiration Coefficient and Productivity of Transpiration in Summer Wheat *Hordeiforme* 0189 under Different Conditions of Water Supply

Series of experiment	To the whole mass		To the seed bulk	
	Transpiration coefficient	Productivity of transpiration	Transpiration coefficient	Productivity of transpiration
Control, optimal 60% moisture throughout development . . .	512	1.95	1337	0.75
Hardening at the outset of tillering	638	1.57	2156	0.46
Hardening during stalk-formation	432	2.31	2125	0.47

to changes under the action of hardening. A great ratio of small bundles containing developed elements of xylem is encountered in leaves of hardened plants. Earlier hardening results in a stronger development of xylem in small bundles and reduces their significance as accumulators of assimilated products. We may thus conclude that hardening, in «overcharging» the leaf with water supplying vascular bundles, leads to an excessive expense of water (low productivity of transpiration) and cannot therefore secure an intensive synthesis of organic matter. On the contrary, it results in a drop of the yield because the structural changes it induces inhibit the outflow of assimilates and thus lower the energy of assimilative processes. When resorting to irrigation in practical farming, one must endeavour optimal water supply and mineral nutrition, this being an indispensable condition of inducing such structural changes in plants as would secure their maximum productivity.

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LIGNUM TECHNOLOGY

AFTER-EFFECT STRAIN AND «LIMIT OF PLASTIC FLOW» OF WOOD

By G. M. IVANOV

(Communicated by A. A. Richter, Member of the Academy, 30.III.1938)

The phenomena of the after-effect in wood due to mechanical influences are almost entirely unexplored. Neither in the USSR nor in other countries any works concerning this problem were published except that by Belian-kine (1) who established the so-called «limit of long time resistance» of wood, as a stress at a zero rate of flow.

These problems of a great scientific and practical importance were being studied by the author during recent four years (beginning from 1934). The most essential results of these investigations are summarized in this paper.

We stated that at relatively low stresses, other conditions being constant, rapidly damping strains of elastic after-effect take place. For higher stresses we found a continuous increment of strains. Thus, beginning with a certain stress, σ_n, undamping strains (or creep strains) develop which have the features of a constant rate plastic flow.

The value σ_n may be determined from the curve of relation between the rate of the after-effect strain and the stress. Such a curve of the average rate at the initial period (from 2 to 4 minutes after loading) was obtained for the case of a transverse bending of pinewood ($10\times10\times667$ mm specimen), the load being applied by degrees (Fig. 1). Up to the stress $\sigma_n \approx 850$ kg/cm^2 there is a linear relation and afterwards the rate of flow sharply increases. In this case σ_n makes nearly 80% of the ultimate strength, equal to 1100 kg/cm^2.

The presence of a similar break in the curve of the after-effect strain rate (Fig. 2) was stated for the case of a simultaneous effect of bending and bearing of oakwood ($12\times54\times75$ mm Derewiagine's plates, average initial rate during the first 2.5 min.). The value σ_n is nearly 60% of the breaking effort equal to 3570 kg. The stress σ_n at which the plastic flow starts can be called the «limit of plastic flow». Beyond the limit of plastic flow the rate of flow also has a linear relation to the stress. The physical nature of the limit of plastic flow is to be sought for in the properties of cellulose, the main body of the mechanical tissue of wood, i. e. of secondary layers of cellular walls (2). It is quite evident that plastic flow of a constant rate may result only from a sliding of micellae, i. e. from a micellar flow peculiar to heterogeneous bodies, and consequently to cellulose (3).

What is the nature of the residual strain which develops in wood with the beginning of loading, i. e. with $\sigma < \sigma_n$?

In order to elucidate this problem the values of residual and elastic strains were compared. The results obtained (Table) left no doubt of the existing proportionality. Indeed, the significance is quite satisfactory, since the statistical index of precision is 0.2 and 1.7%.

The similar nature of the residual and the elastic strains is confirmed by the results of studying the swelling of artificial fibers (5). We refer to the fact that in the course of a succeeding swelling, the residual strain obtained by means of a preliminary extension of the fiber in a dry state, disappears

Transverse bending (4)

Load, kg	Strain, mm		Ratio of residual strain to elastic strain, %	Deviations	
	elastic	residual		δ	δ^2
24.6	0.94	0.02	2.13	+ 0.8	0.64
49.2	1.88	0.04	2.13	+ 0.8	0.64
73.8	2.83	0.06	2.12	− 0.2	0.04
98.4	3.78	0.08	2.12	− 0.2	0.04
123.0	4.73	0.10	2.11	− 1.2	1.44

$M = 2.122$ $\quad \Sigma\delta^2 = 2.80$

$\sigma = \pm\sqrt{\frac{\Sigma\delta^2}{n-1}} = \pm 0.84$ $\quad p = \pm\frac{\sigma}{\sqrt{n}M}100 = \pm 0.2\%$

Nailed connection

Effort, kg	Strain, mm		Ratio of residual strain to elastic strain, %	Deviations	
	elastic	residual		δ	δ^2
76	0.027	0.015	55.5	—	—
152	0.076	0.024	31.6	+ 2	4
228	0.168	0.052	31.0	− 4	16
304	0.305	0.100	32.8	+ 14	196
380	0.495	0.150	30.3	− 11	121

$M = 31.4$ $\quad \Sigma\delta^2 = 337$

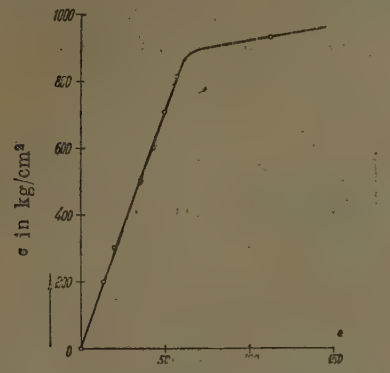
$\sigma = \pm 10.6$ $\quad p = \pm 1.7\%$

entirely. This residual strain is reversible by its nature and only an internal friction prevents a whole reversal of strains when the action of forces is removed. Thus, according to the nature of strains the wood deformation process may be subdivided into two distinctly different zones: zone I, for $\sigma < \sigma_n$, which will be characterized by a reversible nature of strains, i. e. a zone of an imperfect elasticity, and zone II, for $\sigma \geqslant \sigma_n$, characterized by the presence of irreversible strains, resulting from a micellar flow, or a zone of plastic (or creep) strains.

The absence of a micellar flow at the beginning of loading, i. e. the circumstance that $\sigma \neq 0$, evidently results from the initial orientation of micellae of cellulose. It is quite evident that the value of the «limit of plastic flow», i. e. σ_n, is essentially determined by the degree of the micellar orientation of cellulose, as well as the mechanical properties of the latter and those of wood (6). And since the secondary layers of cellular walls present an interlacement of helically twisted fibrillae (7),

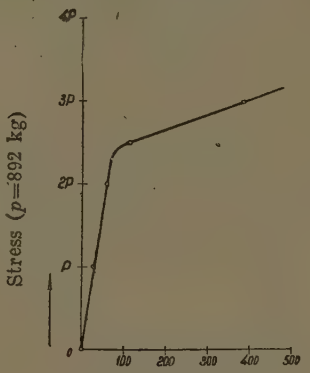
any force effect is taken up by cellulose in the direction of its initial orientation, and therefore, up to a certain value of stresses, in this case up to σ_n, some initial rigidity will take place which will be characterized by an absence of the micellar flow.

Similar features of the after-effect are evidently natural for a series of heterogeneous bodies having an initial orientation of micellae; it supplies



In thousandth mm/min

Fig. 1.



In thousandth mm/min

Fig. 2.

a basis for a development of another, more accurate method of appreciating their mechanical properties according to the beginning of a micellar flow.

However, the absence of a micellar flow for stresses lower than σ_n is not an indication of a complete absence of a strain increase at the action of a constant stress. Experiments have shown that in the case of varying moisture and temperature* and in the zone of an imperfect elasticity, i. e. up to the «limit of plastic flow», a continuous strain increase occurs.

This increase does not take place uniformly, following with a certain lag the variation of the above-mentioned factors. Consequently, a sliding of micellae is here out of the question, since it occurs only beyond the limit of plastic flow, yet an irreversible displacement of micellae may be stated here.

Any cycle of moisture variation produces a shifting of the liquid phase particles. It is evidently more probable that these, as well as the micellae, will each time assume a somewhat different position, corresponding to the vectorial effect of outside forces. These variations in the position of the micellae will be extremely insignificant, but they will still result in an increment of irreversible strains. The temperature variations would produce the same effect. This insignificant value of the strain increment up to the limit of plastic flow was evidently the reason why Beliankine (1) has established a certain stress limit corresponding to a zero rate of flow, though in reality the strain increment ought to take place at any value of the stress, since Beliankine's experiments were carried out under usual laboratory conditions, i. e. at varying temperature and moisture.

This stress *oc* (Fig. 3) for $v=0$ was obtained by Beliankine by means of an extrapolation (till intersection with the ordinate axis) of the straight line, *ab*, of the second section of the diagram $\sigma-v$ beyond the limit of plastic flow. In doing so, the initial section, *oa*, was omitted, which led to an erroneous conclusion. Practically, we may mention here only an approximate

* I. e. under actual conditions of service of wooden structures.

determination of the limit of plastic flow, since $oc \approx \sigma_n$. The latter statement is confirmed by the fact that all «time—strain» graphs obtained by Beliankine are inclined straight lines (at the second period of a stationary rate of flow) indicating a constant rate of flow, i. e. a plastic flow, which may take place only beyond the limit of plastic flow.

Let us proceed to the so-called «limit of long time resistance» of wood. The latter is determined ([1]) as an ordinate of the experimental curve asymptote (Fig. 4), showing a relation between the value of the constantly acting stress and the period of time passing up to the moment of rupture.

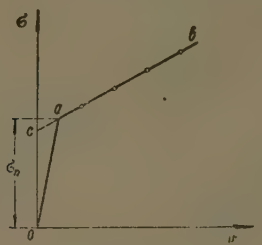


Fig. 3.

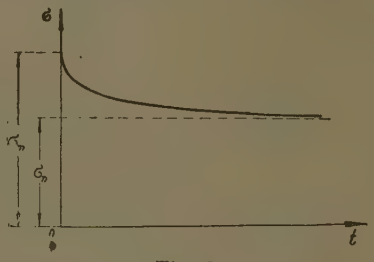


Fig. 4.

The conclusion drawn as to the real significance of the ordinate, *oc*, in the graph for $\sigma - \nu$ (Fig. 3), approximately equal to the limit of plastic flow, permits of discovering the physical nature of the value of the so-called l i m i t o f l o n g t i m e r e s i s t a n c e, equalized by Beliankine to the ordinate *oc*. It is evident that this limit is in reality nothing else than the limit of plastic flow, i. e. t h e s t r e s s c o r r e s p o n d i n g t o t h e b e g i n n i n g o f a m i c e l l a r f l o w.

Consequently, it may be considered established that t h e p r e s e n c e a n d t h e a m o u n t o f t h e p l a s t i c o r c r e e p s t r a i n (i. e. t h o s e o f a m i c e l l a r f l o w) a r e a p r i n c i p a l c r i t e r i o n, d e t e r m i n i n g t h e p e r i o d o f t i m e d u r i n g w h i c h a s t r u c t u r a l m e m b e r w i l l c a r r y a g i v e n l o a d t i l l t h e m o m e n t o f r u p t u r e.

Owing to a coincidence between the value of the limit of plastic flow and that of the so-called «limit of long time resistance», the first expression, i. e. the limit of plastic flow, is to be accepted as a more general one and correctly presenting the nature of the phenomenon. In many cases the second expression entirely loses its purport, e. g. in the case of a creep strain of wood (bearing stress, nailed connections, etc.), where the idea of a breaking force has no sense whatever.

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ENTWICKLUNGSMECHANIK

ÜBER DIE ROLLE DES SCHILDDRÜSENHORMONS IN DER FEDERBILDUNG VON HÜHNERN UND TAUBEN

Von A. A. WOITKEWITSCH

(Vorgelegt von I. I. Schmalhausen, Mitglied der Akademie, d. 29.III.1938)

Wir verfügen heute über ein beträchtliches experimentelles Material hinsichtlich der Rolle der Hormone bei den Vorgängen der Morphogenese. Die ersten Entwicklungsetappen dieser Richtung charakterisieren sich durch Versuche allgemein schematischer Erfassung der Abhängigkeit der formbildenden Prozesse von diesen oder jenen Komponenten des endokrinen Systems, wobei die aufgestellten Schemata im weiteren Umwandlungen erleiden.

Als Beispiel hierfür sei auf die Analyse des Zusammenhanges der sekundären Geschlechtsmerkmale im Federkleide der Vögel mit der hormonalen Funktion der Gonaden hingewiesen. Pezard (12), Zawadowsky (4) u. a. stellten fest, dass die Kastration des Hahnes den Charakter der Entwicklung des Gefieders im wesentlichen nicht verändert, während das Gefieder der Henne sich nach Entfernung des Eierstockes in der Richtung des Männchens verändert. Die sich daraus ergebende Schlussfolgerung über die ausschlaggebende Rolle der Gonaden in der Entwicklung der Geschlechtscharaktere lässt sich, wie später erwiesen wurde, nicht auf alle Vogelarten erweitern. So besteht z. B. bei den *Passeres* kein Zusammenhang zwischen den Gonaden und den sekundären Geschlechtsmerkmalen des Federkleides [Keck (6), Nowikow (10)].

Ähnliche Beispiele liessen sich auch hinsichtlich der Schilddrüsenfunktion anführen.

Auf Grund der Versuche mit Verfütterung von Schilddrüsensubstanz an Vögeln (Tauben), die zuerst von Carlson mit Mitarbeitern (5) angestellt, später an sehr grossem Material von Parhon (11), Giacomini (2), Zawadowsky (3) und Križenecky (7) fortgesetzt wurden, gelangte man zu der Vorstellung, dass die Entwicklung des Gefieders durch das Schilddrüsenhormon bestimmt wird. Weitere experimentelle Tatsachen zwangen indessen zur Einschränkung der Gültigkeit dieser Schlussfolgerung. So beobachteten z. B. Lektorsky und Kusmina (9) in Versuchen an thyreoidektomierten Küken die Mauser auch in Abwesenheit von Schilddrüse.

Weitere Analysen zeigten, dass nicht nur der Federausfall (bei der Mauser), sondern auch die Entwicklung der neuen Generation keinesfalls so eng von der Funktion des Schilddrüsenapparates abhängt, wie früher angenommen wurde. Die von uns [Woitkewitsch (1)] durchgeführten Versuche über mehrfache Aktivierung der Federbildung mit paralleller funktioneller Erschöpfung der eigenen Schilddrüse der Vögel zeigten, dass die Federn verschiedener Pterylien auf das Schilddrüsenhormon verschieden reagieren. Die

für den Flug ausschlaggebenden Flügel- und Schwanzfedern sind z. B. imstande bei ungenügender Menge oder bei völliger Abwesenheit des Schilddrüsenhormons zu wachsen, während die kleinen Rumpffedern sich unter diesen Bedingungen nicht entwickeln, denn sie zeigen eine engere Abhängigkeit von der Schilddrüsenfunktion. Diese verschiedene Reaktion der Federpapillen verschiedener Pterylien auf das Schilddrüsenhormon erweist sich also als eine während der Phylogenese entstandene Eigentümlichkeit. Der Zusammenhang der Morphogenese und der Funktion des endokrinen Systems (speziell des Schilddrüsenapparates) kann sich also bei verschiedenen Tierarten verändern, was durch die späteren Versuche bestätigt werden konnte. Es verändert sich hierbei nicht nur der Grad des Zusammenhanges zwischen der Drüse und den Formbildungsprozessen, sondern gleichzeitig damit auch der Charakter der Drüsenentwicklung, die Intensität des morphogenetischen Vorgangs usw.

Die Vögel scheiden sich hinsichtlich des Charakters der postembryonalen Entwicklung scharf in zwei Gruppen und werden demgemäss eingeteilt in Nestvögel und Brutvögel.

Die ersteren sind zu der Zeit, wenn sie aus dem Ei kriechen, nur schwach differenziert (z. B. Tauben), während die letzteren sofort nach dem Auskriechen zu selbständigem Leben befähigt sind (z. B. Hühner). Diese beiden Gruppen zeigen, wie bekannt, einen sehr verschiedenartigen Charakter des allgemeinen Wachstums und der Entwicklung. Das Wachstum eines Täubchens verläuft sehr intensiv und ist nach 35—40 Tagen abgeschlossen, bei den Kücken braucht dagegen dieser Vorgang eine Zeitspanne von einigen Monaten. Auch die Intensität der Entwicklung des Federkleides zeigt entsprechende Unterschiede. Es ist deswegen durchaus natürlich zu erwarten, dass die Schilddrüsen der jungen Tauben im Zusammenhange mit dem intensiven allgemeinen Wachstum und der Differenzierung des Organismus beträchtliche Veränderungen ihrer Entwicklung erleiden müssen. Wir unternahmen eine histologische Untersuchung und Bestimmung der biologischen Aktivität der Schilddrüsen von jungen Tauben (vom Zeitpunkt des Auskriechens aus dem Ei bis zum Alter von 35 Tagen) und Kücken (vom 11-ten Tage der Bebrütung bis zum Alter von 2 Monaten). Die erhobenen Befunde zeigten, dass die Entwicklung des Schilddrüsenapparates einen der Gesamtentwicklung des Organismus ähnlichen Charakter trägt. Bei den Vertretern der Nestvögelgruppe (Tauben) ist das junge Individuum im Augenblick des Auskriechens aus dem Ei schwach differenziert, wobei das gleiche auch hinsichtlich der Schilddrüse zu sagen ist. Bei den Hühnern (Brutvögelgruppe) erlangt die Schilddrüse einen analogen Entwicklungsgrad schon am 13—14 Tage des embryonalen Lebens. Die biologische Aktivität der Schilddrüse, die eine gesetzmässige Abhängigkeit von der Struktur des Organs aufweist, erleidet während der weiteren Entwicklung einen bestimmten Zyklus von Veränderungen, die bei den Vertretern der beiden Gruppen—bei den Kücken und jungen Tauben—in der Hauptsache einen gemeinsamen Charakter besitzen. Die Periode der Differenzierung des Organs wird sowohl bei den Kücken als auch bei den jungen Tauben von einer allmählichen Vergrösserung der biologischen Aktivität begleitet. Die Drüse gelangt alsdann in eine Phase erhöhter Sekretion. Diese Erscheinung fällt mit der Periode der grössten Intensität der Entwicklung des Organismus zusammen. Sogar ein so wichtiges Element der allgemeinen Differenzierung, wie die Entwicklung des Federkleides, wird bei den Vertretern beider Gruppen von einer Phase verstärkter Exkretion der aktiven Substanzen aus der Schilddrüse begleitet. Bei den sich intensiv entwickelnden jungen Tauben ist diese Erscheinung sehr klar und in kurzer Zeit beendet, während bei den Kücken, deren Entwicklungsperiode eine längere ist, die Kurve der Veränderung der Drüsenaktivität keine scharfen

Sprünge aufweist, sodass die Erhöhung der Sekretion nicht so bedeutend ist.

Der Unterschied in dem Grade des Zusammenhanges der Federbildung mit der Schilddrüsenfunktion lässt sich in viel stärkerem Masse bei erwachsenen Individuen demonstrieren, d. h. zu einer Zeit, wo die mit dem allgemeinen Wachstum zusammenhängenden Vorgänge, bei denen die Rolle des Schilddrüseninkrets zur Genüge bekannt ist, schon abgelaufen sind.

In Versuchen an erwachsenen Vögeln aktivierten wir eine massenhafte Entwicklung einer neuen Federgeneration durch Auszupfen des gesamten Federkleides. Zu verschiedenen Zeitpunkten nach dem Beginn der Aktivierung bis zum vollständigen Abschluss des Federwachstums wurden mikroskopische Untersuchungen der Schilddrüsen der Versuchsvögel und parallelle biologische Testierungen des Thyreoideagewebes vorgenommen. Strukturelle Veränderungen der Drüse fanden auch hier, wie früher, entsprechenden Ausdruck in Schwankungen der biologischen Aktivität; dies erlaubte eine maximal objektive und vollständige Beurteilung der Veränderungsdynamik der Funktion des Schilddrüsenapparates bei unseren Versuchsvögeln.

Bei Tauben erfährt die Funktion des Schilddrüsenapparates beim massenhaften Federwachstum eine scharfe Veränderung. Die Dimensionen des Organs nehmen stark zu; die histologische Struktur zeigt verstärkte Exkretion der aktiven Substanzen aus der Drüse, was auch durch die Daten über die biologische Aktivität des Drüsengewebes bestätigt wird. Bei den Tauben wird also die Zeit der intensiven Federbildung von einer starken Hyperfunktion der Schilddrüse begleitet.

Bei Hühnern (weisse Leghorn) lassen sich nach einer ähnlichen Aktivierung des Federwachstums keine wesentlichen Veränderungen in der Tätigkeit des Schilddrüsenapparates beobachten. Bei der mikroskopischen Untersuchung der Schilddrüsen von Hühnern (zu verschiedenen Zeiten vom Beginn des Wachstums der neuen Federgeneration an) finden wir keine strukturellen Veränderungen der Schilddrüse, die auf eine aktive Exkretion hinwiesen.

Das Wachstum des Gefieders, wie auch das Gesamtwachstum, verläuft bei den Tauben bedeutend schneller als bei den Hühnern. Larionov ([8]) konnte zeigen, dass die Geschwindigkeit des Federwachstums bei den Tauben den analogen Koeffizienten der Hühner um das zweifache übersteigt. Es ist möglich, dass die verschiedene Reaktion der Schilddrüsen von Hühnern und Tauben mit der ungleichen Geschwindigkeit des Federbildungsvorgangs zusammenhängt. Gleichzeitig damit ist es möglich, dass der Grad der Beteiligung des Schilddrüsenhormons an dem Federbildungsvorgang bei Hühnern und Tauben ein verschiedener ist; die Unabhängigkeit des Federbildungsprozesses von der Schilddrüse ist bei den Hühnern wohl eine weitgehendere als bei den Tauben; bei den letzteren kann man von einer Unabhängigkeit der Federbildung nur hinsichtlich einiger bestimmter Pterylien sprechen.

Auf die möglichen Erklärungen des Mechanismus, der die Entwicklungsprozesse des Federkleides mit dem endokrinen Komplex verbindet, wollen wir hier nicht eingehen, müssen aber doch ganz speziell hervorheben, dass der Charakter und der Grad dieses Zusammenhanges bei relativ nahestehenden systematischen Gruppen, wie sie z. B. durch Hühner und Tauben vertreten werden, stark variieren kann. Bei den Tauben ist der Zusammenhang der Federbildung mit der Schilddrüsenfunktion während der postembryonalen Entwicklung sehr deutlich ausgesprochen. Ebenso deutlich lässt sich dieser Zusammenhang auch bei erwachsenen Individuen bei künstlicher Aktivierung der Federentwicklung demonstrieren. Bei den Hühnern, bei denen wir während der postembryonalen Entwicklung Veränderungen in der Schilddrüsenfunktion wahrnehmen, die wir sowohl dem Gesamtwachstum als auch

der Federbildung zuschreiben können, lassen sich bedeutende Veränderungen der Schilddrüsentätigkeit nicht beobachten. Sie fehlen auch bei dem experimentell provozierten massenhaften Wachstum des Gefieders.

Die Tatsache, dass eine gegebene Komponente des endokrinen Systems bei Vertretern von zwei Vogelgruppen verschieden auf denselben morphogenetischen Prozess reagiert, wird also keinesfalls durch Einzelversuche gedeutet, sondern durch eine folgerichtige Erforschung der Gesetzmässigkeiten der natürlichen Entwicklung des Schilddrüsenapparates unter Berücksichtigung der Phylogenese (deren Momente in der allgemeinen Form im verschiedenen Charakter der allgemeinen Entwicklung des Organismus ihren Ausdruck gefunden haben).

Die angeführten Daten zeigen die Unmöglichkeit eines universellen Schemas der kausalen Abhängigkeit der Morphogenese von dem endokrinen System. Gleichzeitig damit zeigte unser Material, wie mannigfaltig sich die gegenseitige Beziehung der (lokalen und zentralisierten) Komponenten des Formbildungsprozesses gestalten kann, bedingt von den Besonderheiten der allgemeinen Entwicklung der Vertreter von nur zwei systematischen Gruppen.

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DEVELOPMENT MECHANICS

BITS OF LIVER LUNG RESTORED TO LIFE UNDER THE SKIN OF AN AXOLOTL

By N. N. SINAKEVITCH

(Communicated by N. V. Nassonoff, Member of the Academy, 1. IV. 1938)

A number of authors have observed the transplanted bits of the liver [Mitsuda (1), Herzheimer (2), Holtfreter (3), Ribbert (4)].

These authors transplanted bits of liver into various parts of the body; under the skin, in the lymphatic glands and the abdominal cavity [Wangensteen (5)]. All these experiments were performed on mammals: dogs, rabbits, guinea pigs and rats. The authors established that after a certain time, all the transplantates undergo degeneration and decay in which processes the elements of the connective tissue of the host are involved. Later on a new formation of the liver tissue takes place, either at the expense of the epithelium of bile ducts (Ribbert) or at the expense of the central liver cells, not affected with degeneration (Herzheimer). Moreover, the formation of liver cells and that of bile ducts proceed independently. Holtfreter transplanted the liver anlage to the embryos of the *Urodela* and obtained it involved in the life process of the host which later on resulted in the development of the liver.

Acad. N. V. Nassonoff inserted bits of liver under the skin of the leg of adult axolotls. In one case, after a period of 324 days, he could establish on sections that the transplanted bit had grown together with the surrounding tissue. Moreover, on the insertion of bits of a lung under the skin of the tail of axolotls, he noticed three cases of fusion of the inserted bits with the surrounding tissues. In all cases the blood-system of the host was connected with that of the transplantate. No additional formations were observed (6).

This material consisting in a series of sections was passed to me for investigation. The material was fixed after Susa and dyed according to Mallory.

On that account I was unable to establish the presence of glycogen and other admixtures in the protoplasm of the liver cells.

A histological examination of the sections showed that a bit of liver inserted under the skin of the tarsus of an axolotl was not dissolved by the phagocytic elements and was not deteriorated, as it had been observed in a number of cases. Favourable conditions were probably created on insertion and the bit of liver set out in life under the skin. Studying the sections, we see the transplantate (Fig. 1) surrounded by a thin loose layer of connective tissue containing vessels. On the top it is covered with skin, whose structure remains undamaged. In its lower part the transplantate borders with striated muscles of the extremities. The first section under the cutis contains small groups of liver cells, situated in the form of separate liver lobes and separated

by wide layers of loose connective tissue containing venous vessels. The blood capillaries reach the lobes and are distributed over their surfaces closely touching the liver cells. We have not noticed any sign of degeneration in the liver cells. All of them have a normal appearance (Fig. 2).

The cells are either globular or polyhedral in shape. The protoplasm is granulous, the nucleus round with a well defined structure. Binucleal cells are of frequent occurrence. On subsequent sections the number of lobes increases, the layers of the connective tissue become thinner, and the blood capillaries more numerous. In the sections made through the central part of the transplanted bit, we found the typical structure of a normal liver of an axolotl. The liver lobes lie very closely, the blood capillaries running along their periphery form a thick capillary net. The bile capillaries may be seen

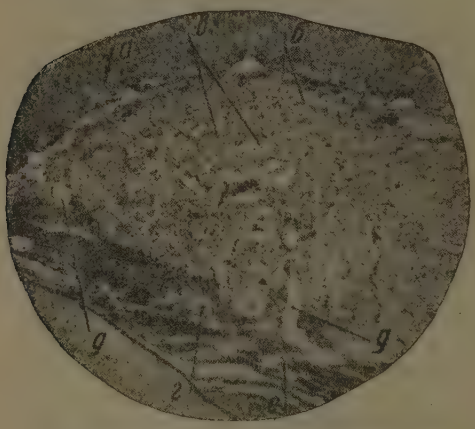

Fig. 1. A section of a liver transplantate. 324 days after a subcutaneous transplantation. Fixation Susa, dyed after Mallory. Slight magnification, *a*—epithelium, *б*—the liver tissue, *в*—venous capillaries, *г*—arteries, *д*—vein, *е*—muscles.

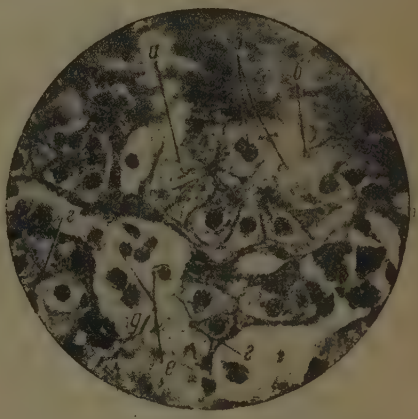

Fig. 2. Section of a liver transplantate under the skin of an axolotl. Immersion 1/12. *а*—liver lobes, *б*—liver cells, *в*—the nuclei of the liver cells, *г*—bile capillary, *д*—venous capillary, *е*—erythrocytes.

in the centre of the lobes. Rather large lumps of pigment are seen in the layers of connective tissue between the lobes. The connective tissue is rich in various cellular elements among which lymphocytes prevail. Lymphoid accumulations are also met. The blood capillaries contain a great number of erythrocytes. As we have already mentioned the whole transplantate is surrounded by loose connective tissue which contains venous and arterial vessels. Liver lobes lying separately from the transplantate are also noticeable in this connective tissue. These separate lobes are also accompanied by blood capillaries and in the centre of the lobes we may also notice bile ducts. Comparatively large venous branches reach the transplantate on both sides. Reaching the transplantate the veins are branching, some of them enter the liver from the lateral side and some go to the connective tissue under the cutis giving birth to smaller branches which enter the liver on the upper side. All these venous vessels, on entering the transplantate, form a net of capillaries which covers the liver lobes. On the lower side venous vessels from the intramuscular connective tissue enter the transplantate, attended by very small arterial branches. The latter run along the intralobular layers of the connective tissue and seem to enter the large net of the venous capillaries. Unfortunately, it is not possible to establish from the sections which of the venous

vessels reaching the transplantate carry the blood in and which of them carry it out.

A graphic reconstruction showed only the fact that the venous vessels which approach the transplantate from all sides gradually join together to form a capillary net. There is no doubt that blood circulation had been organized in the transplantate, this circulation being performed at the expense of the venous vessels, while the arteries present single small branches forming no independent capillary net. We have to assume that the transplantate was acting under the skin of the leg of an axolotl.

Insertions of bits of lung under the skin of an axolotl gave three cases where the transplantate took root in the life of the host. I received material obtained at various dates, 139, 229 and 330 days after the insertion of bits of lung un-

Fig. 3. A section of lung transplantate made 139 days after a subcutaneous transplantation on the tail of an axolotl. Slight magnification. Fixation Susa, dyed after Mallory. *а*—epithelium, *б*—connective tissue, *в*—glands, *г*—transplantate of the lung, *д*—muscles, *е*—alveoli.

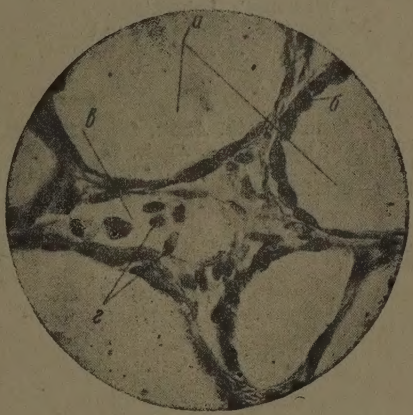

Fig. 4. Section of a lung transplantate under the skin of the tail of an axolotl. Immersion 1/12. *а*—alveoli, *б*—epithelium, *в*—capillary, *г*—erythrocytes.

der the skin of the tail of an axolotl. In these preparations we see a piece of the lung parenchyme lying directly under the skin (Fig. 3). A thick connective tissue of the cutis is undamaged and touches the transplantate in upper part. Below and on the sides a loose subcutaneous connective tissue surrounds the transplantate. This connective tissue contains arterial and venous vessels which together with the surrounding tissue enter the lung parenchyme. In the transplantate we see the respiratory and air containing section of the lung of an axolotl. We must point out to the normal character of the epithelium lining the inner surface of the air containing and respiratory sections. The cells of the flat respiratory epithelium of the alveoli contain large oval nuclei. The structure of the nuclei is expressed well. The protoplasm of the cells adheres to a thin membrane. The blood carrying capillaries lie directly under the membrane (Fig. 4). In the lumen of the capillaries erythrocytes may be seen. With the transition of the respiratory sections into air containing ones the epithelium becomes higher, first being of cubic character and later of a low cylindrical one. The cells of the cubic and cylindrical epithelium possess ciliae on their free surface. Between the cylindrical cells there are cup shaped cells filled up with secretion. The connective tissue between the

alveoli and other sections of lung contains bundles of non-striated muscles and vessels. Preparations of a later period, made after 229 and 330 days, present the same picture as those made after 139 days. We could notice no signs of degeneration in the epithelial cells. The structure of the respiratory and air containing sections and the relation between the vessels and the lung parenchyme is the same. In all the three cases, all sections of the lungs are stretched which is due to the tissue liquid having accumulated inside the transplantate.

In all the sections we could trace that the respiratory and air containing ducts penetrating the transplantate end there in blind. All parts of the lung damaged before insertion regenerated and formed closed cysts and tubules. In all three cases blood circulation had been completely restored.

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